

Cable Route Planning in Complex Environments Using Constrained Sampling

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Abstract

We present a route planning algorithm for cable and wire layouts in complex environments. Our algorithm precomputes a global roadmap of the environment by using a variant of the probabilistic roadmap method (PRM) and performs constrained sampling near the contact space. Given the initial and the final configurations, we compute an approximate path using the initial roadmap generated on the contact space. We refine the approximate path by performing constrained sampling and use adaptive forward dynamics to compute a penetration-free path. Our algorithm takes into account geometric constraints like non-penetration and physical constraints like multi-body dynamics and joint limits. We highlight the performance of our planner on different scenarios of varying complexity.

Keywords: motion planning, adaptive dynamics, articulated bodies, cable route planning

1 Introduction

Designing cable and wire layouts is often a complex and tedious process in building construction. Incorrect cable and wire layouts can be costly due to poor planning and may require significant modifications and design reviews. Current practices include scaled prototypes onto which the cable (or wire) layout is constructed by using route planning algorithms for motionless rigid cable segments. CAD systems are often used to assist the process during the early design stage. In these systems, the paths of the cables or wires are first calculated, then rigid segments are put through these paths to simulate cable or wire layout. However, current CAD systems do not take into account the cable's or wire's dynamics properties and their interaction with the environment. It should be noted that in some applications dynamic, or even quasi-static, simulation may not be necessary. But, while actually laying a cable or simulating the process, forces such as friction or gravity may influence the path taken to arrive at the goal configuration. For instance, in pulling or pushing a long cable toward a goal configuration, the friction of the cable with a passage or corner may influence what path is taken. Or, if an interactive visualization for rapid prototyping is required, it would be important to model the bending or sagging of the cable between supports due to gravity. In these cases, dynamics is necessary to generate physically plausible paths, configurations, and motion.

To the best of our knowledge, there is very little or no work done on cable (or wire) routing that includes both realistic physical simula-

tion and motion planning. Cable and wire routing can be posed as a robot motion planning problem to compute the path for a highly articulated and deformable robot, i.e. the cable (or the wire). However, the dimensionality of the configuration space of such a deformable robot is very high. In addition, the simulation of highly articulated and deformable bodies can be expensive and time consuming.

1.1 Main Results

In this paper, we present a novel approach for cable routing in complex environments. We assume that the cable can be modeled as a highly articulated robot with multiple links and many degrees of freedom (dofs).

Our route planning algorithm is based on a variant of PRM that samples near the surfaces of the obstacles in the workspace. It initially generates random samples at the corners and edges of the environment. If a path cannot be found, it samples on the obstacles by using a ray shooting approach. The resulting global roadmap computed this way lies near the *C*-obstacles in the configuration space, also known as the *contact space* [Latombe 1991]. This roadmap is then used as the guiding path to plan the motion of the entire cable. During the simulation phase, whenever a collision occurs between the first link of the cable and the environment, the initial path is adjusted based on “constrained sampling”, which recomputes a new node near the contact space. Our method expands the initial roadmap from a node belonging to the contact space (computed from a contact point) and finds a new node around the neighborhood of the contacting node using the projection method [Wilhelmson 1976]. In order to simulate cable dynamics, we use an adaptive forward dynamics algorithm [Redon et al. 2005] with quasi-static numerical integration that selects the most important joints to perform bounded error dynamic simulation in a hierarchical manner. The quasi-static integration removes unnecessary oscillation due to inertia which helps provide smoother robot motion. Moreover, we develop efficient collision handling techniques to resolve the contacts for the remaining links during the simulation.

Our algorithm achieves realistic simulation results and performs each simulation step at interactive rates. We have implemented and tested our system on Alienware AMD AthlonTM 64 X2 Dual-Core with 2.41 GHz Processor 4800++ and 2GB of RAM. Our algorithm can simulate a path for cables consisting of 200-300 links (or dofs) at interactive rates (averaging about 17 ms per simulation step) in modestly complex environments with tens of thousands of polygons. Our adaptive dynamics algorithm provides significant performance gain compared to the prior linear-time forward dynamics algorithms. In addition, our constrained sampling technique generates effective and useful samples near the obstacles, such as the walls, doorways, and around other support structures.

1.2 Overview

The rest of the paper is organized as follows. In Section 2, we briefly survey the prior work on motion planning for deformable robots, cable simulation and cable routing. We give an overview of our approach in Section 3. We present our cable simulation algorithm based on adaptive forward dynamics in Section 4. We describe our planning algorithm in Section 5 and highlight its performance in Section 6.

2 Previous Work

In this section, we give a brief overview of prior work in cable simulation, cable routing and motion planning for deformable objects.

2.1 Cable Simulation

Cable simulation has been studied in the recent years. Hergenrother and Dahne present an algorithm for the real-time simulation of virtual cables [Hergenrother and Dahne 2000]. Their simulation is based on inverse kinematics. They model the cable by using consecutive cylinder segments that are connected by ball joints. Given the start and goal positions of the cable, their algorithm calculates the shape of the cable by considering energy minimization. In practice, their algorithm is fast and applicable to interactive applications. However, their approach does not simulate the dynamics of the cable and does not perform collision handling, which is essential for realistic applications.

Looock and Schomer describe an application of rigid body simulation to assembly tasks in virtual environments and extend their system to real-time simulation of deformable cables [Looock and Schomer 2001]. They model the cable as a chain of rigid segments. The cable simulation is achieved by using mass-spring model with generalized springs. They use stiff linear springs and torsion springs to preserve the length of the cable.

Gregoire and Schomer present a numerically stable and physically accurate simulation tool for one dimensional components in [Gregoire and Schomer 2006]. They model the bending and torsion using the Cosserat model and they use a generalized spring-mass system with a mixed coordinate system for the simulation. However, their simulation is slow for modeling long cables. For example, the simulation time for cable that has 300 points is around 128 ms per step.

2.2 Cable Routing

The design of a cable harness is a complex and costly process. Many approaches have been proposed to automate the design process. Conru [Conru 1994] describe a system to route a cable harness using genetic algorithms. The genetic algorithms are used to search for routes which are close to the global minimum. The cable harness routing problem is decomposed into generating a harness configuration and computing a route for the harness in the environment. Both of these problems are solved using genetic algorithms.

Holt et al. [Holt et al. 2004] present a virtual reality system to aid cable harness designers. Their approach focuses on usage of human engineer's knowledge in the design process. Their goal is to provide an engineer with interactive tools to design a human-in-the-loop system.

Both of these approaches do not consider the physical properties of the cable. They assume that the cable is composed of rigid segments and the number of segments can be adjusted by the user.

2.3 Motion Planning for Articulated and Deformable Robots

Motion planning is a well studied problem in robotics. Most of the work has been on rigid or articulated robots with a few degrees of freedom. Our variant of PRM and constrained sampling is similar to the OBPRM algorithm [Amato et al. 1998] and local planning in the contact space [Redon and Lin 2005], although our realization is quite different. In addition, we incorporate physical and mechanical constraints using adaptive forward dynamics and contact handling, whereas prior algorithm mostly deal with geometric constraints such as non-penetration.

There is relatively less work on motion planning for deformable objects. Some of the earlier work on deformable robots included specialized algorithms for bending pipes [Sun et al. 1996], cables [Nakagaki and Kitagaki 1997] and metal sheets [Ngugen and Mills 1996]. Holleman et al. [Holleman et al. 1998] present a probabilistic planner capable of finding paths for a flexible surface patch, modeled as a low degree Bèzier patch, using an approximate energy function to model deformation of the part. Guibas et al. [Guibas et al. 1999] describe a probabilistic algorithm for a surface patch, modeled as the medial axis of the workspace. Anshelevich et al. [Anshelevich et al. 2000] present a path planning algorithm for simple volumes such as pipes and cables by using a mass-spring representation. Lamiroux et al. [Lamiroux and Kavraki 2001] propose a probabilistic planner capable of finding paths for a flexible object under manipulation constraints. The object's deformation is computed by using the principles of elastic energy from mechanics which makes the motion planning difficult for handling the end constraints and finding minimum energy curves. In [Moll and Kavraki 2004], a different curve parametrization technique is used for handling low-energy configurations. In addition, contact points with simple obstacles are considered in finding a minimal energy curve configuration. However, finding the exact contact points that makes the curve at minimum energy is still a difficult task. Bayazit et al. [Bayazit et al. 2002] describe a two-stage approach that initially computes an approximate path and then refines the path by applying geometric-based free-form deformation to the robot. Gayle et al. [Gayle et al. 2005] present an algorithm for path planning for a flexible robot in complex environments. The algorithm computes collision free paths based on physical and geometric constraints. The collision detection between deformable object and the environment is accelerated by using graphics processors. Saha et al. [Saha and Isto 2006] present a motion planning technique for the manipulation of deformable linear objects. The application of their method in self-knotting and knotting around simple static objects by using coordinating dual robot arms is illustrated in the paper. The motion planning algorithm depends on the geometrical model of the deformable object and the robot arms. It does not consider any physical properties of the deformable linear object. Many of these algorithms exploit geometric properties and often do not consider the physical constraints of the robot, such as collision detection and contact handling. Some recent approaches deal with general deformable robots and environments [Rodriguez et al. 2006], but do not model friction and motion constraints (e.g. joint limits) which could be necessary to realistically model the interaction between the cable and the rest of the environment.

3 Overview

In this section, we give an overview of our planning algorithm. We introduce the notation used in the remainder of the paper and present our framework to solve motion planning as a constrained dynamical system and model the robot as an articulated chain with a high number of dofs.

3.1 Notation

We assume that each cable can be modeled as a sequence of m rigid bodies connected by $m - 1$ 2-dof revolute joints. The joints are implemented as two 1-dof revolute joints, one of which is rotated 90 degrees about the central axis of the cable. The configuration, $C(t)$, of the cable at time t can be described as a vector of joint angles along with the position and orientation of the base of the robot. The position of the head (first link) of the cable is represented by $C_{head}(t)$, which is the position and orientation of this link.

We assume that the set of obstacles are rigid and they are represented as $O = \{o_1, o_2, \dots\}$ in the workspace, W . A roadmap, $G = \{V, E\}$, in the free workspace, the space external to the obstacles, consists of a milestones $V = \{v_1, v_2, \dots\}$ and links $E = \{e_1, e_2, \dots\}$. A path in this roadmap is an acyclic sequence that connects two milestones.

Problem Formulation: The problem can be stated as follows: Find a sequential set of cable configurations $C(t_1), \dots, C(t_f)$ such that no $C(t_i)$ intersects with any obstacle in O , and $C(t_i)$ is near the obstacles, where $C(t_1)$ and $C(t_f)$ are the initial and final configurations of the robot (cable) respectively.

3.2 Cable Simulation

When simulating a cable, there are a number of considerations which must be taken into account. First, to model a realistic cable, we assume that its length will not change during the simulation. This holds since little or no stretching will occur in most wires and cables when forces are applied to the ends.

In addition, our simulation must preserve geometric and mechanical properties of the cable itself. This accounts for penetrations with the environment and also with itself as well as preventing a cable from bending too much. To efficiently simulate the cable, we take advantage of adaptive forward dynamics for articulated bodies [Redon et al. 2005] and develop a new collision handling method. Since many constraints can introduce unwanted oscillation in the robot motion due to large penalty response forces, we use a quasi-static variation of Euler integration in order to advance the simulation.

Our planner applies several constraints to the cable in order to ensure that it can successfully reach its goal. Path constraints are used to move it along a specified path, and collision constraints ensure that it is penetration free and also enforces joint angles.

3.3 Motion Planning

Our planning algorithm builds upon constraint-based motion planning (CBMP) proposed by [Garber and Lin 2002].

This approach involves two main stages; a roadmap generation stage and an execution stage. The first stage is largely done as a precomputation step and is responsible for finding a *guiding path* through the environment. This guiding path does not need to be completely collision free with respect to the robot. The second stage uses constrained dynamical simulation to move the robot along this guiding path toward the goal. This allows the robot to locally adapt the guiding path to its own structure.

There are several advantages to the CBMP approach. Guiding path generation is both efficient and simple, while the path itself also automatically ensures that both geometric and physical constraints are preserved. For our problem, this allows the cable to move along a simple guiding path without violating its constraints.

4 Cable Simulation

The geometric and physical characteristics of the cables should be simulated realistically and efficiently for real-time applications. In this section, we first state our assumptions and the basic representation for modeling cables, then we describe our approach for simulating cable dynamics.

4.1 Assumptions and Representations

Cables have various material properties depending on its type and usage. In this work, we assume that a cable conserves its length during the simulation. A cable consists of rigid segments that cannot be stretched or sheared. Its length remains constant when dragged or pulled. Its cross section is undeformable and the mass of each segment is same along the entire cable. To model each cable, the length of the cable can be specified by the number of rigid segments or the length of each rigid segment. By changing one of these parameters, the user can model cables of varying lengths. Based on these assumptions, we can model each cable as a chain of articulated linkages, i.e. as a highly articulated robot.

The optimal algorithm for computing the dynamics of a kinematic chain takes linear time [Redon et al. 2005]. To approximate a cable well using a highly articulated chain would require many linkages. This computation can become rather costly for long cables. Therefore, we use the *adaptive forward dynamics* algorithm [Redon et al. 2005] for cable simulation. The main advantage of this approach is that it lazily recomputes the forward dynamics of a cable by only simulating the joints that best approximate the overall motion of the entire chain with bounded errors. The adaptive dynamics automatically selects active joints, or those which will be simulated, based on motion error metrics to compute an error-bounded approximation of the articulated-body dynamics. The user can change the motion error metric for controlling the number of joints that will be active or rigidified during the simulation.

The contact handling is also an important issue in cable simulation. The cable should deform realistically due to contacts and the penetration should be prevented when there is a collision between the cable and the nearby obstacles. We develop a penalty-based contact response capable of handling joint limits and other external forces along with a fast collision detection method to achieve sub-linear-time collision handling for multi-body systems.

Next we will describe the basic components of the adaptive multi-body dynamics and contact handling for our cable simulation.

4.2 Articulated-body dynamics

The adaptive dynamics algorithm is built upon Featherstone's *divide-and-conquer algorithm* (DCA) [Featherstone 1999a], [Featherstone 1999b]. Featherstone's algorithm is a linear time algorithm to compute the forward dynamics of an articulated body based on the forces applied to it. The algorithm relies on the following *articulated-body equation*:

$$\begin{bmatrix} \hat{\mathbf{a}}_1 \\ \hat{\mathbf{a}}_2 \\ \vdots \\ \hat{\mathbf{a}}_m \end{bmatrix} = \begin{bmatrix} \Phi_1 & \Phi_{12} & \cdots & \Phi_{1m} \\ \Phi_{21} & \Phi_2 & \cdots & \Phi_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{m1} & \Phi_{m2} & \cdots & \Phi_m \end{bmatrix} \begin{bmatrix} \hat{\mathbf{f}}_1 \\ \hat{\mathbf{f}}_2 \\ \vdots \\ \hat{\mathbf{f}}_m \end{bmatrix} + \begin{bmatrix} \hat{\mathbf{b}}_1 \\ \hat{\mathbf{b}}_2 \\ \vdots \\ \hat{\mathbf{b}}_m \end{bmatrix}, \quad (1)$$

where $\hat{\mathbf{a}}_i$ is the 6×1 spatial acceleration of link i , $\hat{\mathbf{f}}_i$ is the 6×1 spatial force applied to link i , $\hat{\mathbf{b}}_i$ is the 6×1 bias acceleration of link i (the acceleration link i would have if all link forces were zero), Φ_i

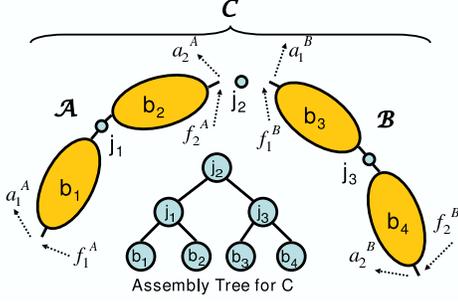


Figure 1: Construction of an articulated body. An articulated body \mathcal{A} is connected to body \mathcal{B} at the principal joint, j_2 , to form body \mathcal{C} . The assembly tree for \mathcal{C} is shown beneath the body. Forces and accelerations which govern \mathcal{C} 's motion are shown.

is the 6×6 inverse articulated-body inertia of link i , and Φ_{ij} is the 6×6 cross-coupling inverse inertia between links i and j .

The DCA employs a recursive definition of an articulated body: an articulated body is a pair of articulated bodies connected by a joint. The forward dynamics of the articulated body are computed in essentially two steps: *main pass* and *back-substitution pass*. When the DCA completes, all joint accelerations and all kinematic constraint forces are known.

In order to advance the state of the simulation, a quasi-static variation of Euler integration is used. In this variation, the body is considered to be at rest throughout the integration step. Thus, both angular and translational velocities are zero and integration only considers the net acceleration.

4.3 Adaptive articulated-body dynamics

Featherstone's DCA is linear in the number of joints in the articulated body. Determining a path or resolving contacts for a highly articulated body could be prohibitively slow using a typical forward dynamics algorithm. In order to improve the performance of the planner, we incorporate the adaptive dynamics algorithm by Redon *et al.* [Redon *et al.* 2005] to lazily simulate the articulated body motion that best represents the overall motion of the robot with an error-bounded approximation. Essentially, this enhanced algorithm allows us to systematically choose the appropriate number of joints that are simulated in the articulated body, by automatically determining *which* joints should be simulated, in order to provide a high-quality approximation of the articulated-body motion.

4.4 Collision Handling

The collision detection between the cable and the environment is performed using a hybrid bounding volume hierarchy. We perform two culling steps, based on *axis-aligned bounding boxes* (AABBs) and *oriented bounding boxes* (OBBs), to help localize potential collisions, before performing intersection tests at the triangle level. We pre-compute and store one hierarchy of oriented bounding boxes for each rigid cable link and each rigid environment obstacle. We also precompute one axis-aligned bounding box for each obstacle in the environment. The OBB hierarchies and AABBs of environment obstacles do not have to be updated during planning.

At runtime, we determine the intersections between the cable and the environment obstacles using the following collision detection

algorithm:

- **AABB hierarchies update:** For each mobile rigid link, we determine a bounding AABB using the root OBB of the OBB-Trees [Gottschalk *et al.* 1996]. We then compute an AABB for each node of the assembly tree (*i.e.* for each subassembly of the articulated body) using a bottom-up pass. We thus obtain one AABB hierarchy per articulated body, whose structure is *identical to the assembly tree of the articulated body*.
- **AABB culling:** Using the AABB hierarchies, we detect potentially colliding rigid links and objects.
- **OBB culling:** When two rigid objects are found to potentially intersect after the AABB culling step, we recursively and simultaneously traverse their OBB hierarchies to help localize potential collisions between pairs of triangles [Gottschalk *et al.* 1996].
- **Triangle/triangle intersection tests:** Whenever two leaf-OBBs are found to overlap, a triangle/triangle intersection test is performed to determine whether the triangles contained in the leaf-OBBs intersect. When they do, we report the corresponding intersection segment.

After collisions between the cable and the obstacles have been determined, the collision response is computed using a penalty-based approach as described in Sec. 4.5.2.

4.5 Constraint Forces

In our simulation, along with internal forces for kinematic constraints, we apply external constraint forces for making the cable move along the computed path and for handling interaction between the cable and the obstacles. Next, we will describe the formulations of these constraint forces.

4.5.1 Attraction Force to the Goal

To make the rest of the cable follow the computed path, we apply attraction forces to the first link of the cable along the direction of the path. This force is basically a spring force between the first link of the cable and the computed path.

$$\mathbf{f}_i^{Attraction} = k(\mathbf{L}_i - \mathbf{L}_{rest}) \quad (2)$$

In this formulation, k is the spring constant, \mathbf{L}_i is the distance between the path and the first link of the cable, and \mathbf{L}_{rest} is the intended distance between the first link of the cable and the path.

4.5.2 Contact Handling Forces

When a collision is detected between the cable and the obstacle, we apply a force in the direction of contact normal to keep the cable apart from the surface of the obstacle. Let x_i be the position of contact link i , x_i^{proj} is the projection of x_i on the obstacle and N_i is the normal vector at position x_i^{proj} . Then the collision response force will be proportional to the penetration distance d , where

$$\mathbf{d} = (\mathbf{x}_i^{proj} - \mathbf{x}_i) \cdot \mathbf{N}_i \quad (3)$$

and the contact handling force will be

$$\mathbf{f}_i^{Contact} = kN_i \mathbf{d} \quad (4)$$

where k is the collision coefficient.

Given the basic steps of our cable simulation module, we will next describe our motion planning algorithm and show how our cable simulation is used in our cable route planning.

5 Motion Planning for Cable Layouts

Our cable route planning algorithm is composed of three key phases:

1. Global Roadmap Generation
2. Guiding Path Estimation
3. Path Adjustment and Constrained Dynamics Simulation
 - (a) If there exists a collision between the first link and the environment, compute the next milestone using constrained sampling and modify the path accordingly;
 - (b) For the remaining links, simulate the cable motion using adaptive forward dynamics with efficient contact handling and positional constraints.

Next, we will describe each step in more detail.

5.1 Global Roadmap construction

The probabilistic roadmap (PRM) algorithm is commonly used for robot motion planning. We develop a variant of PRM for computing the roadmap of the environment. For simplicity and efficiency, we first treat the cable as a point robot and then make path adjustments using constrained sampling and constrained dynamics simulation. This strategy makes the roadmap construction fast and effective.

5.1.1 Contact-Space Sampling

One of the important aspects of any randomized roadmap planner is the method used to generate samples in the configuration space. Since typically cables and wires should be placed in such a way that they attract as little attention as possible, one of our sampling goals is to find samples in the free space which are close to the obstacles (e.g. on or near the wall) and near the corners of the buildings.

We achieve this goal by systematically finding “corners” of a large structure. This is done by first computing the intersections of three or more constraint planes (walls), then the intersections of two constraint planes. Random samples are first taken at these locations. These points are shifted away from the walls by an amount δ to ensure that the local planning can construct the roadmap of the environment. After generating the samples near the corners, we construct our roadmap and we check whether a valid path exists. If there is no viable path from the initial to the goal configuration, then extra milestones are generated by shooting rays on these constraint planes (walls), which typically are the xy -, yz -, or xz planes for a building.

5.1.2 Local Planning

Another important step in PRM is local planning which connects collision-free nodes. We use a distance-based approach for finding the edges in the roadmap from a milestone v_i . All milestones that are within some search radius of v_i are placed into a neighbor set, N . For each milestone v_j in N , we determine if there is a straight

line segment between v_i and v_j that does not intersect with any obstacle in O . For connecting the initial and final configuration to the roadmap, we also place them in the milestone set and perform local planning for them as well.

5.2 Computing a Guiding Path

Once we have generated a roadmap that provides the desired amount of coverage, we use it to generate paths between some given start and goal configurations. We first link the start and goal configurations to the roadmap. In particular, we add the start and goal configurations to our graph as query nodes and create edges between these nodes and all other nodes reachable from them. After these nodes have been added we can execute any graph search to find a path from the start to the goal. We use Dijkstra’s shortest path algorithm with distance values as weights for computing the initial path.

5.3 Simulation and Runtime Path Modification

Once an estimated path is computed, we begin our simulation loop with the robot in its initial configuration. To make the cable follow the path during the simulation, we apply our attraction force to the first link of the cable in the direction of the path. This makes the first link of the cable move along the direction of the path, however the rest of the cable does not always follow the approximate path due to the dynamics of the cable. In order to constrain the motion of the entire cable, we apply additional path constraint forces to a subset of the links of the cable which are selected based on their position with respect to the curved regions of the path segments followed by the first link of the cable. This approach is similar to fixing portions of the cable against a wall to ensure it remains there.

As the cable traverses the path, we check the collisions between the cable and the obstacles. If there exists a collision between the cable and the obstacles, we apply collision handling algorithm to avoid the penetrations. In addition, if there is a collision between the first segment of the cable and the obstacles, we update the current path segment by computing a new, constrained milestone as described in Sec. 5.4.

For each simulation or path-query step, we apply the following algorithm:

1. Apply attraction force (Sec. 4.5.1) to the first link of the cable;
2. Sum all forces and deform the cable using adaptive forward dynamics (Sec. 4.3);
3. Perform collision detection (Sec. 4.4) between cable and obstacles;
4. Apply contact response (Sec. 4.5.2) to resolve the collisions;
5. If there exists a collision between the first segment of the cable and the obstacle, update the current path segment between v_i and v_{i+1} .
 - (a) Compute new milestone v_{new} using constrained sampling (the approach is presented in section 5.4);
 - (b) Update the position of milestone v_{i+1} to v_{new} ;
 - (c) Check collision between the new path segment v_i - v_{i+1} and the environment.

We repeat steps (1) - (4) until a path is found between the initial and final configurations, v_i and v_f . If no path is found, it is reported and the user can either add more milestones or conclude that there is no

safe path. Otherwise, we end the simulation when it is reported that the object is sufficiently close to the final configuration.

5.4 Constrained sampling

Given the contact information between the first segment of the cable and the obstacle, we can determine the local tangent space of the obstacles for a given pair of sampled nodes near the obstacles. We use this tangent space to constrain the search of a new node when we found an intersection between the first link of the cable. and the obstacles.

Assume v_i is a point on the path and an external force toward this point is applied to the robot. Let $P = (P_x, P_y, P_z)^T$ denote the Cartesian coordinates of the contact point in the world frame. The contact point P corresponds to a point on the first link of the robot, and is a 3-vector $P(C_i)$ depending on the configuration C_i of the cable and the environment. Let $n = (n_x, n_y, n_z)^T$ denote the Cartesian coordinates of the contact normal in the world frame, and assume that this normal is directed toward the exterior of the obstacle. The local non-penetration constraint is:

$$dP(C_i) \cdot n \geq 0, \quad (5)$$

where $dP(C_i)$ is the small variation in the position of P corresponding to a small variation $d(C_i)$ around the configuration C_i of the robot. This non-penetration constraint defines a polyhedral set of valid variations. Provided the robot in the configuration C_i is in a consistent state with no interpenetration, the valid variations set is non-empty.

Because we use linear constraints, the set of valid variations is only a local piecewise-linear characterization of the valid space around v_i . This set of valid variations can be efficiently used for local planning. At runtime, whenever there exist a contact between the first link of the robot and the obstacle, we use the set of valid variations to help determine a new node v_{new} , by projecting the intentional variation on the set of valid variations. More specifically, whenever a new tentative node v_t is randomly chosen in the neighborhood of the contact-space node, we perform the following operations before checking its validity with a discrete collision checker:

1. Compute the corresponding tentative variation $dv_t = v_t - v_i$.
2. Project the tentative variation dv_t on the set of valid variations to obtain the new variation dv .
3. Set the new milestone $v_{new} = v_i + dv$

The projection of the intentional variation dv_t onto the polyhedral set of constraints is performed by using the Wilhelmsen projection algorithm [Wilhelmsen 1976].

Finally, we note that the combination of constrained sampling in the contact space and sampling by ray shooting allows us to accumulate obstacle constraints: whenever a new node v_{new} is determined by extending a node v_i , we check whether the near-obstacle constraints of v_i , are still valid in the new configuration v_{new} . This occurs for example when the robot comes in contact with an obstacle.

6 Results

In this section, we present the results of our approach on three different benchmarks of varying complexity. Below are our benchmark scenarios:

- **Bridge Model-** This model consists of over 5,000 polygons and the cable is represented as a 280-link robot with 5,000 polygons.

- **House Model-** This model consists of over 14,500 polygons and the cable is represented as a 280-link robot with 5,000 polygons. The house has four rooms and there is only one entry to the house. The house does not have a ceiling.
- **Building Model-** This model consists of over 18,200 polygons and the cable is represented as a 280-link robot with 5,000 polygons. The building has seven floors and each floor has different shapes and dimensions. In order to illustrate the cable simulation and planning visually, we place the cable around the outside of the building.
- **Car Model-** This model consists of over 19,668 polygons and the cable is represented as a 280-link robot with 5,000 polygons. This model is obtained from the MPK model database.

Our algorithm runs at interactive rates, averaging about 60 fps. Table. 1 illustrates the timings of our planning and sampling technique in detail.

We also benchmark our algorithm by changing the sampling technique in our simulation. Instead of constrained sampling, we use random sampling in 3D. In constrained based sampling, we generated samples near the corners, edges and walls, and we applied constraint sampling during the simulation. However, in random sampling the samples are randomly generated in 3D. It is important to note that each sampling step takes longer but ensure no collision between samples using our constrained sampling. When using random sampling, each step takes much less time, but many samples generated are not necessarily in free space, thus can result in longer simulation time to compute a collision free path. In this benchmarking experiment, we use our adaptive dynamics simulation framework and collision handling method in both cases for cable simulations. Table. 2 shows the timing comparison of two sampling techniques. In all cases, the precomputatin using constrained sampling takes roughly same amount of time as random sampling. But, the simulation time is less using roadmaps computed from our constrained sampling method.

To demonstrate the path quality generated by our algorithm with constrained sampling in the contact space vs. using random sampling for only the first segment of the cable in our approach, we show a comparison image highlighting the difference in path quality for the House in Fig. 2. Notice that constrained sampling results in better cable placement than random sampling for cable route planning. The computed path using constrained sampling tend to stay around the corners, the edges, and walls as real cables should; while the computed path using random sampling may be a valid one but can be obtrusive or unsafe for human operating in the same work space.

It is also important to note that we cannot apply standard PRM directly as the planner must know the final configurations of all cable segments, in order to find the intermediate configuratons for the entire cable, which has a very high degrees of freedom and can take a very long time to plan any path. The final configuration of the entire cable is what we also aim to compute at the same time in this paper. And, the final configuration of all cable segments can be automatically computed and simulated using our integrated approach presented here.

We also include additional sequences taken from our simulation runs (see Fig. 3). Our cable route planning algorithm is able to automatically compute realistic paths for these modestly complex models. Our algorithm using constrained sampling systematically finds milestones which are either on or near to the obstacles (walls, corners, structural supports, etc.) and the cable moves along the path naturally using our adaptive multi-body dynamics at interactive rates. Please refer to the project website,

Scene	# of Tri. in Cable	# of Tri. in Environment	Total joints	Motion Error Metric	Avg. Time Dynamics (s)	Avg. Time Simulation (s)	Total Sim. Time (s)
Bridge	5,600	5,000	280	0.01 %	0.0026	0.0045	27.03
				0.1 %	0.0019	0.0040	23.67
				0.5 %	0.0013	0.0034	20.07
House	5,600	14,500	280	0.01 %	0.0030	0.0051	49.98
				0.1 %	0.0022	0.0048	47.04
				0.5 %	0.0017	0.0041	40.01
Building	5,600	18,200	280	0.01 %	0.0031	0.0051	41.31
				0.1 %	0.0025	0.0047	38.07
				0.5 %	0.0019	0.0040	32.40
Car	5,600	19,668	280	0.01 %	0.0034	0.0055	32.49
				0.1 %	0.0020	0.0040	23.66
				0.5 %	0.0016	0.0037	22.23

Table 1: Performance for different benchmarks

Scene	Total joints	Motion Error Metric	Constrained Based Sampling		Random Sampling	
			Roadmap+Query Time	Total Simulation Time	Roadmap+Query Time	Total Simulation Time
Bridge	280	0.5 %	12.31	20.07	8.82	26.82
House	280	0.5 %	36.42	40.01	53.26	49.86
Building	280	0.5 %	41.64	32.40	39.74	42.93
Car	280	0.5 %	46.21	22.23	43.25	26.23

Table 2: Comparison between Constrained Based Sampling (CBS) and Random Sampling

http://gamma.cs.unc.edu/CABLE_PLAN

for more path sequences generated by our cable route planner.

7 Conclusion

We present a novel algorithm for planning of cable layouts in complex environments. We propose a variant of PRM using a novel constrained sampling coupled with a fast adaptive forward dynamics algorithm and efficient collision handling. Our approach is applicable to many applications, such as wire routing. We have demonstrated our algorithm on four interesting scenarios and the initial results are rather promising.

Our current approach has some limitations. We first perform constrained sampling for the head of the cable and then the rest of the cable follows by adaptively computing its forward dynamics with contact handling for the entire cable. This algorithm does not guarantee physically-accurate motion at all times, but an error-bounded approximation. Nevertheless, the resulting path is always kinematically valid and collision free.

There are several directions for future investigation. It is possible that we can tightly integrate the adaptive forward dynamics framework with constrained sampling. This may result in a more robust system, but at higher computational costs. We can apply continuous collision detection to the cable simulation during the path following.

We would also like to explore cable route planning for cables with multiple branches, which can be useful in some real-world applications. In addition, we plan to apply our algorithm to more complex environments, such as a power plant. We would like to further improve the roadmap construction and collision detection for massive models consisting of many millions of geometric primitives. And, we would like to apply our simulation in a wider variety of situations, such as bundled cables, adding supports for a single or multiple cables, and also to include a wider variety of constraints.

Acknowledgement

The authors would like to thank our reviewers for the useful feedback which helped to improve the work. This work was supported in part by a Department of Energy High-Performance Computer Science Fellowship administered by the Krell Institute and ARO Contracts W911NF-04-1-0088, NSF awards 0400134, 0429583 and 0404088, DARPA/RDECOM Contract N61339-04-C-0043 and Disruptive Technology Office.

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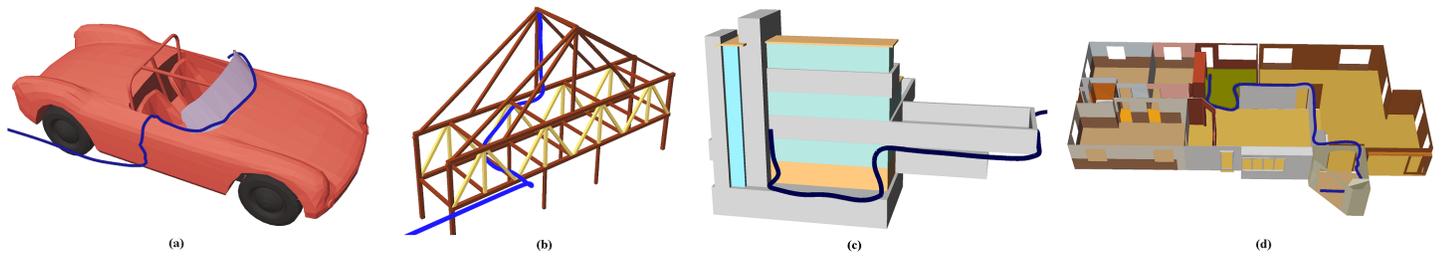


Figure 3: Cable route planning on the (a) Car Model (b) Bridge Model (c) Building Model (d) House Model

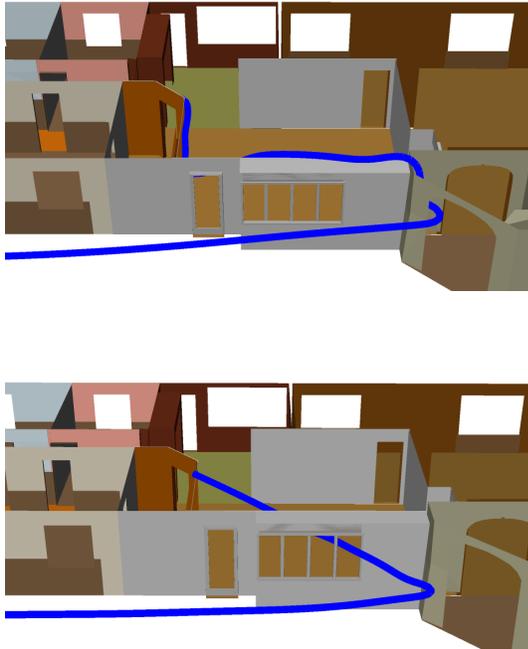


Figure 2: Path quality comparison on the House Model using our algorithm with constrained sampling in the contact space (Top) and with random sampling (Bottom). Notice that the cable is better placed near the support structures on the walls with constrained sampling. Using random sampling, the cable is simply routed from its initial configuration to the final configuration through uncluttered space and the placement of the cable appears rather awkward.

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