# Similar Part Rearrangement With Pebble Graphs 

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#### Abstract

This work proposes a method for effectively computing manipulation paths to rearrange similar objects in a cluttered space. The solution can be used to place similar products in a factory floor in a desirable arrangement or for retrieving a particular object from a shelf blocked by similarly sized objects. These are challenging problems as they involve combinatorially large, continuous configuration spaces. This work proposes graphical tools to quickly reason whether manipulation paths allow the transition between entire sets of object arrangements without having to explicitly store the path for each pair of arrangements. The proposed method also allows to take advantage of precomputation given a manipulation roadmap. The resulting approach is evaluated for scalability and success ratio in simulation for a realistic model of a Baxter robot and executed in open-loop on the real system. ${ }^{1}$


## I. Introduction

Robot manipulators need to be able to rearrange objects in constrained, cluttered human environments. Such a skill can be useful, for instance, in manufacturing, where multiple products need to be arranged in an orderly manner or in service robotics where a robotic assistant, in order to retrieve a refreshment from a refrigerator, must first rearrange other items. This work describes a methodology for solving such tasks in geometrically complex and constrained scenes using a robotic arm. The focus is on the case that the target objects are geometrically similar and interchangeable.

A key challenge for practical rearrangement algorithms is the size of the search space. Problems also become hard when the objects are placed in tight spaces, coupled with limited manipulator maneuverability. This paper deals primarily with these combinatorial and geometric aspects and proposes motion planning methods that return collision-free paths for manipulating multiple rigid bodies.

The approach reduces the continuous space, highdimensional rearrangement problem into several, discrete rearrangement challenges on "rearrangement pebble graphs" (RPGs). The inspiration comes from work in algorithmic theory on "pebble graphs" [1], recent contributions in multirobot motion planning [2], [3], as well as work in manipulation planning [4]. The approach does not limit the type of rearrangement challenges that can be addressed to achieve efficiency, however, it does make certain concessions. For instance, it must be possible to retract the arm to a safe configuration from every stable grasped pose in a solution sequence. Given this requirement for solutions, probabilistic completeness can be argued within this set while also achieving computational efficiency, the same way that it was achieved for multi-robot path planning [2].

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Fig. 1. In order for Baxter to grasp the can at the back of the shelf, the other cans need to be rearranged. Videos, including runs on the real system, can be found here: http://tinyurl.com/mc38zr2

Related Work and Contribution: Rearrangement planning [5], [6] can be viewed from many different perspectives.
Planning among Movable Obstacles: Navigation among movable obstacles (NAMO) is an NP-hard challenge [7]. Thus, most efforts have dealt with efficiency [8] and provide completeness results only for problem subclasses [9]. A probabilistically complete solution was proposed [10], but works only for simple robots (2-3 DOFs).

Manipulation: Manipulation problems can be approached with a "manipulation graph" using sampling-based planners [4]. Manipulation among multiple movable objects has been considered for "monotone" problems where each obstacle can be moved at most once [11]. The current work can resolve "non-monotone" challenges.

Task and Motion Planning: Rearrangement plannin is an instance of integrated task and motion planning [12], which emphasizes the need to reason over the properties of set of states without enumerating them, an important insight of the proposed "pebble graph" approach.

Multi-robot Motion Planning: This work is motivated by progress [13] with "pebble motion on a graph", itself a hard problem, where pebbles must move from initial to goal vertices on a graph [1], [14]. Feasibility can be decided in linear time for such problems [1], [14], inspiring a method for continuous multi-robot motion planning [2]. The method employs sampling-based planners and reduces multi-robot challenges into many discrete pebble problems. The current paper is motivated by this approach and defines "rearrangement pebble graphs" (RPGs) for manipulation challenges.

## II. Rearrangement Planning

One way of solving the unlabeled rearrangement problem would be to build a manipulation graph [15] in the entire state space. This is, however, a high-dimensional space and given that motion planning is hard, efficient solutions cannot be achieved easily as the number of objects increases. The idea


Fig. 2. Average results over 10 trials are shown. The primary axis measures computation time (limited to 600 seconds), while the secondary axis depicts solution quality (before and after smoothing). For both graphs values of $b=1,2,3,4$ have been used. Left: Results for randomly placed objects rearranged into a grid configuration, where $k=2,3,4,6,8$. Right: Results from one of the non-monotone benchmarks with $k=4$.
here is to abstract out the motion of the manipulator and reason directly about the movement of objects between different stable poses. Reasoning about the movement of multiple objects can take place over discrete graphical representations so as to take advantage of linear-time path planning tools for rearranging unlabeled "pebbles" on a graph from an initial to a target arrangement [1].


Fig. 3. Each edge on an RPG is the combination of a reaching, transferring and retracting path.

A sampling approach can be used to define graphs where nodes correspond to stable poses and edges correspond to collision free motions of the arm that transfer an object between stable poses (Fig.3a). If such a graph (Fig.3b) is connected and contains all the poses from the initial and target arrangements, then a discrete solver can be used to define a solution in the continuous space as long as placing objects in different poses does not cause collisions [1].

It may be difficult or impossible, however, to construct a single such graph that directly solves the problem. For example, the poses in the initial and target arrangements could be already overlapping, or it may not be possible to ensure connectivity with collision-free motions of the arm. Motivated by work in the multi-robot motion planning literature [2], the current paper considers multiple such graphs, referred to "rearrangement pebble graphs" (RPGs).

Within each RPG the discrete solver can be used to achieve all feasible arrangements, given its connectivity. If the RPG has one connected component, then all possible arrangements over the graph can be attained for unlabeled objects and they do not need to be explicitly stored. If the RPG has multiple connected components, a signature, which specifies how many objects exist in each connected component, describes the feasible arrangements. It should also be possible to switch between different RPGs, if they share at least $k$ poses that can be occupied by objects, given the corresponding signatures.

This gives rise to a hypergraph structure, where each node corresponds to an RPG and a signature. Edges correspond to transitions between such hypernodes. The initial and target
arrangements define two such hypernodes. Then the approach generates and connects hypernodes until the initial and target arrangement are connected on the hypergraph. At that point, the rearrangement problem is solved and the necessary motions of the manipulator can be extracted along the path connecting the initial and target nodes on the hypergraph.

## III. Evaluation

The proposed algorithm has been evaluated in a series of rearrangement problems. Two different cases of randomly placed objects and two non-monotone problems have been tested. Different values of $k$ objects and $b$ empty nodes in the RPG have been considered in the experiments (Fig.2). The model of the manipulator used corresponds to a 7-DOF Baxter arm, on which open loop trials were run. The tradeoff between the computational benefits and connectivity in an RPG, introduced by $b$, is evaluated.

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[^0]:    ${ }^{1}$ An extended version of this work was submitted to WAFR 2014.

