Trajectory Planning for Robots: the Challenges of Industrial Considerations

Joonyoung Kim and Elizabeth A. Croft, *Member*, *IEEE*

***Abstract*— In this paper, the problem of trajectory planning for industrial robots is discussed. Well-known methods such as the time-optimal motion planning and trajectory smoothing techniques are considered from an industrial application perspective. We argue that existing methods are limited in use if some important considerations are not taken into account such as path accuracy, the importance of minimum-time trajectory, the need to minimize online planning cycle times, and the performance of motion controllers. The interaction between trajectory planning and control is demonstrated in a simple example.**

I. Introduction

Industrial robots are widely used in various industries. In most applications, the accuracy and speed of the robot motion is critical since these factors significantly affect the manufacturing quality and cost. Therefore, motion control for industrial robots continues to be an active research area.

Even though there are a large number of papers related to trajectory planning, there is still an open debate on how to generate a 'good' trajectory. Over the years, researchers have proposed methods such that the resulting trajectories are time-optimal [1]-[5]. In these methods, the dynamics of the robot drives the solution. The major limitation of this approach is that the resulting trajectories cannot be accurately tracked by a generic controller due to the high frequency components of the trajectory generated by switching from max/min torque [1], [2] or torque-rate [3]. The controller must either filter the trajectory and thus leave the desired path, or attempt to track the trajectory resulting in large vibrations. Further, this approach, involving online dynamics computations poses a high computational burden on the planning system. Despite increased processing power, minimizing computation time remains in important consideration for online planning [5] in order to reduce planning cycle time, and reduce hardware and software costs.

Alternatively, some researchers have favoured increasing trajectory smoothness in a trade off with time-optimality [6]-[11], to avoid these high frequency components. In this approach, the trajectories are expressed as the 4^{th} or 5^{th} order polynomials to limit the higher derivatives such as jerk and snap. Even though it has shown that the resulting trajectories could obtain improved path accuracy, the resulting robot motions are sub-optimal since they neglect the robot dynamics. Furthermore, solving for these polynomials can become relatively complicated with nonzero boundary conditions for velocities and acceleration [11], a common scenario for industrial robot applications.

In the document, the limitations of existing methods are discussed based on some of the important considerations for modern industrial robots such path accuracy independent of trajectory speed, the demand for fast motion in industrial applications, the need for online planning and the performance limitations of the controllers.

J. Kim is with the Intelligent Control Research Department, Hyundai Heavy Industries Co., Ltd. and also with the Department of Mechanical Engineering, the University of British Columbia (email: joonyoung.gim@gmail.com).

E. A. Croft is with the Department of Mechanical Engineering, the University of British Columbia (email: ecroft@mech.ubc.ca).

II. Related Work

Shin and McKay [1] and Bobrow *et al*. [2] proposed a method that obtains a time-optimal trajectory along specified path. The resulting trajectory switches between maximum and minimum accelerations. Constantinescu and Croft [3] followed a similar approach as in [1] and [2] but limited the maximum torque-rates. Singer [6] proposed an input shaping technique that filters out the frequency component of a trajectory that excites the system assuming that the natural frequency of the system is known. Meckl and Woods [7] used the piecewise 4^{th} order polynomials and showed that the residual vibration can be minimized if the jerk time is tuned at the natural frequency. Macfarlane and Croft [9] proposed a smooth trajectory that is piecewise 5^{th} polynomial and showed that it can be applied for multiple way-points. Antonelli *et al*. [4] and Kim *et al.* [5] optimized the TVP (Trapezoidal Velocity Profile) by calculating robot dynamics at a limited number of points along the trajectories.

III. Practical considerations

A. Speed-independent Path

One of the most important requirements for industrial robots is to maintain path accuracy regardless of operating speed (Fig. 1).

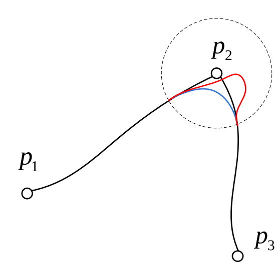

Figure 1. Typical robot paths in case when the trajectory is not planned along the parameterized path (blue line: low speed trajectory, red line: high speed trajectory).

If the path varies when the programed speed changes, there is the potential for collision. This is a serious problem since any robot collision can cause a significant delay in a production line. One method to remove such possibility is to express a path in terms of a single path parameter and then plan a trajectory for the path parameter. For instance, as shown in Fig. 1, if the path is defined in task space, the path $f:\Re \rightarrow SE(3)$ can be expressed as

$$f(s) = \left[\mathbf{p}(s) \quad \mathbf{R}(s) \right] , \tag{1}$$

where, $\mathbf{p} \in \Re^3$, $\mathbf{R} \in SO(3)$, $0 \le s(t) \le 1$, and t is time.

In this way, it is guaranteed that the path is uniquely determined regardless of its trajectories, $s(t)$. However, a significant challenge for this approach is to connect multiple paths both for position and orientation simultaneously, where the associated geometry is nonlinear [12].

B. Demand for high-speed motions

In many industrial applications, the robot's speed is critical. Thus, it is generally required that a robot's trajectory must be optimized at all times. Since the robot's maximum speeds and accelerations vary substantially depending on the robot dynamics, the trajectory design must consider the robot dynamics. However, calculating robot dynamics usually requires a high computational burden. One practical method that alleviates this problem is to use a predetermined trajectory such as a TVP and then solve for the dynamics only at certain points along the trajectory as proposed in [4] and [5].

For example, the method in [4] and [5], can be easily applied to the parameterized path expressed in (1) by solving the equation of motion that is also expressed in the path parameter,

$$\boldsymbol{\tau}(s) = \mathbf{m}(s)\ddot{s} + \mathbf{c}(s,\dot{s}) \in \Re^n , \tag{2}$$

where, $\mathbf{m} \in \Re^{n \times n}$: inertia matrix

$\mathbf{c} \in \Re^n$: velocity dependent terms and gravity

$\boldsymbol{\tau} \in \Re^n$: joint torques

n : the number of independent coordinates.

C. Online trajectory planning

Online planning implies that a robot's trajectory is planned during motion. This ability is important because the motion of industrial robot will often be modified based on operating conditions measured online by external sensors. Thus each trajectory segment must be calculated with non-zero boundary conditions, i.e., non-zero initial and target velocities and accelerations. For an S-curve, there are dozens of different cases that need to be considered depending on the boundary conditions [11]. A trajectory planner that must consider a large number of cases is less desirable from a software management and risk management perspective. The large code base associated with many cases increases the potential for software errors which could lead to a robot collision with an object or a human especially if the trajectory is not required to follow a parameterized path.

D. Relationship with the controllers

It is generally known that smooth trajectories are better in terms of residual vibrations. However, the robotics literature is often unclear on how such smoothness is determined, e.g., identifying the correct tuning for the maximum jerk when smoothing trajectories.

Consider a symmetric TVP, $s_{TVP}(t)$ and a symmetric S-curve, $s_{S-curve}(t)$, assuming the boundary velocities are zero. Then, the frequency responses of these two trajectories can be obtained by Fourier transform.

For a TVP,

$$S_{TVP}(j\omega) = \frac{A}{-j\omega^2}\left(1 - e^{-jT_a\omega}\right) , \tag{3}$$

where, A : acceleration magnitude, T_a : period over which acceleration is applied.

For an S-curve,

$$S_{S-curve}(j\omega) = \frac{J}{-j\omega^3}\left(1 - e^{-jT_j\omega}\right) , \tag{4}$$

where, J : jerk magnitude, T_j : period over which jerk is applied.

The above equations demonstrate why an S-curve in general generates smaller vibrations than a TVP. The power transferred by an S-curve to the system is inversely proportional to the cube of the frequency whereas that of a TVP is inversely proportional to the square of the frequency. However, it can be also observed that the transferred power by a trajectory depends not only on the magnitudes of accelerations and jerks, A and J, but also the time durations, T_a and T_j.

Fig. 2 shows a simulation result that demonstrates the relation between the commanded motion profiles and system response. In the simulation, it is assumed that the robot has 2DOF and each flexible joint is controlled by a PD controller. It is assumed that the natural frequencies of the closed-loop system are 7 Hz and 21 Hz near the tested position.

From the figure, it is evident that the TPV generates larger residual vibrations than S-curves. However, between the two S-curves, the one with a smaller jerk generates a larger vibration than the one with a larger jerk. This is because the jerk of the former S-curve is tuned the natural frequency of the system, thus suppressing the residual vibrations [7]. This implies that increasing smoothness in a trajectory may not always guarantee accurate tracking if the performance of the closed-loop system is not considered.

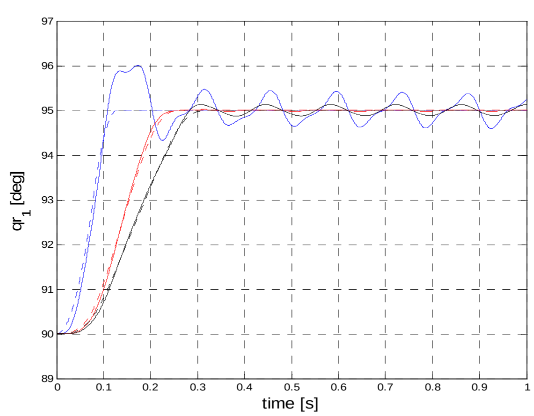

Figure 2. Simulation results of S-curve trajectories with different maximum jerk values (dotted lines: planned trajectories, blue line: TVP, red line: S-curve jerk tuned at the lowest natural frequency of the robot at 7 Hz, black line: jerk tuned at 5 Hz).

IV. CONCLUSIONS

In this paper, the problem of trajectory planning is discussed from an industrial point of view. An open discussion is required regarding how to obtain a 'good' trajectory for industrial robots. The simulation shown suggests that trajectory smoothing alone cannot achieve improved tracking motion performance for industrial robots.

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