Implementing a GPU-Efficient FFT

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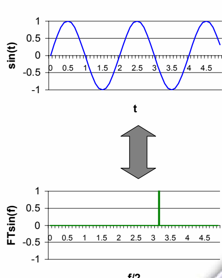
Why Fast Fourier Transform?

- “Classic” algorithm
- Computationally intensive
- Useful
  - Imaging
  - Signal analysis
  - Procedural texturing

What is a FFT?

- Fourier transform
  - Transform function from spatial- to frequency-domain
  - $H(f) = \int_{-\infty}^{\infty} h(t)\, e^{2\pi i f t}\, dt$
- Inverse Fourier transform
  - $h(t) = \int_{-\infty}^{\infty} H(f)\, e^{-2\pi i f t}\, df$



Discrete Forms for Series of Samples

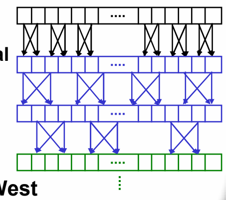
- Discrete Fourier transform
  - $H_n = \sum_{k=0}^{N-1} h_k\, e^{2\pi i k n/N}$
- Inverse discrete Fourier transform
  - $h_k = 1/N \sum_{n=0}^{N-1} H_n\, e^{-2\pi i k n/N}$

Solving Fourier Transforms

- As matrix equation:
  - $H_n = \sum_{k=0}^{N-1} W^{nk} h_k$
  - $\hat{H} = W \cdot \hat{h}$
  - $O(N^2)$ operations
- Recursive (Fast Fourier Transform):
  - $F_k = \sum_{j=0}^{N-1} e^{2\pi i j k/N} f_j$

    $= F_k^e + W^k F_k^o$
  - $O(N \log N)$ operations

Fast Fourier Transform Implementations



- [Numerical Recipes in C]
  - Loop over elements for bit-reversal
  - Loop log N times to recombine neighbors
  - Weights are computed iteratively
- Fastest Fourier Transform in the West
  - http://fftw.org
  - Optimized for current CPU architectures
  - Adapts itself to current CPU cache sizes

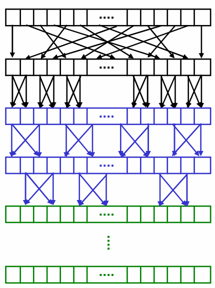
Application Example: SETI@home

- SETI@home Pulse Search
  - Search for dispersed pulses of intrinsically short duration, e.g., pulsars
- Computation task at hand:
  - Have ~2.5 years of data
  - Need to examine every .8ms of that data
  - Each examination requires ~0.34 GFlops
    - mostly in the form of FFTs
  - ~33,507,000,000 GFlops computation
- Needs every help it can get

GPU FFT Feasibility

- 2048 element FFT requires
  - ~8 * 2048 * log(2048) = ~180 KFlops
  - 2048 * 8 = 16KB of data
- Computational limits for GeForceFX 5900 (NV35)
  - Vertex: .450 GHz * 3 units * 4 FLOPS/vector = 5.4 GFLOPS
  - Pixel: .450 GHz * 4 units * 12 FLOPS/unit = 21.6 GFLOPS
  - Total: 27 GFLOPS
- Theoretical times for GPU
  - Download: 16k @ 2.0 GB/s = 8 us (AGP 8X)
  - Computation: 180KFlop @ 27 GFLOPS = ~7 us
  - Upload: 16k @ 0.18 GB/s = 90 us (PCI)

FFT Algorithm Overview



Pass 0: Bit Reversal

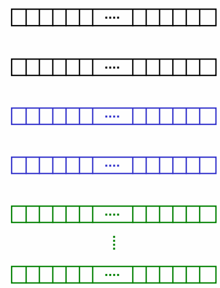
Pass 1: combine 1-neighbors

Pass 2: combine 2-neighbors

⋮

Pass log N: combine N/2-neighbors

Mapping Data-Structures to GPU



1D texture (from AGP)

1D float texture (render target)

1D float texture (render target)

1D float texture (render target)

⋮

1D float texture (render target to be read back to system memory)

GPU Algorithm Overview

- **Download FFT data to GPU as a 1D texture**
  - **2k by 1 texels big**
- **Render quad into float texture render-target**
  - **Quad is 2k pixels wide and 1 pixel high**
  - **Use x pixel position to index texture**
- **Bit-Reversal done as:**
  - **Pass address of pixel as texture coordinate**
  - **Fragment(x) = tex(bitreversal(x))**
  - **Bitreversal() is simply texture look-up**

GPU Algorithm Overview (cont.)

- **Log N combination passes**
  - **Fragment(x) = tex(index0(x)) +
w(index1(x)) * tex(index1(x))**
  - **w(), index0(), and index1() are textures**
    - **Different for every pass**
    - **Pre-computed**
- **Read final render-target back into system memory**

Red Flags for GPU Performance

- 1 + log N passes
  - All data stays on GPU (good)
  - Per-vertex computations trivial (good)
  - Lots of API calls for CPU to instruct GPU what to do
  - GPU has to finish each pass before next one starts
- Only 1D textures
  - GPUs highly optimized for 2D textures
- Complex number computations
  - Complex numbers are 2D
  - But hardware is optimized for 4-vectors

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Batching Many FFT Transforms

- Download 2D texture of coefficients
  - Compute hundreds of FFTs per pass
  - Cuts driver calls by hundreds of times
  - Fully utilizes multi-pipe fragment processing hardware
- Basically uses the same fragment programs
  - Only differ in needing a 2^{nd} texture coordinate

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Using Vector Operations

- Store 2 complex numbers per texture
  - (t0.r, t0.g) is first number
  - (t0.b, t0.a) is second number
- Store 4 complex numbers in 2 textures
  - (t0.r, t0.g, t0.b, t0.a) are real parts
  - (t1.r, t1.g, t1.b, t1.a) are imaginary parts
  - Code is more symmetric
  - But more temporaries are used

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Real World Performance

- CPU
  - FFTW algorithm
  - 3.0 GHz Intel Pentium 4
  - 2048 FFT takes 12 us
  - 1.5 GLOPS
- GPU
  - Algorithm outlined here
  - NVIDIA GeForceFX 5900 Ultra (NV35 @ 450 MHz)
  - 2048 FFT takes 16 us (32 us with readback over PCI)
  - 1.1 GLOPS (.6 GFLOPS with readback)

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Optimization Possibilities

- Range and precision of computation and results
  - Is 16-bit floating point sufficient for registers?
  - Conversion to lower precision has double benefit:
    - Faster to compute
    - Faster to transfer back to CPU
- If range and precision of input is limited
  - Don't compute results, but rather…
  - Replace N passes with table look-up
- Tap into over 5 GLOPS of unused vertex processing

Conclusions

- GPU useful now as co-processor to CPU
- Keep the faith!
  - Faster access to (and particularly from) graphics subsystem is critical, but coming soon
  - GPU parallelism outstripping that of CPUs
  - GPUs will continue to enjoy an advantage over CPUs in dedicated memory bandwidth

Future Work

- Integrate more of the Pulse Search problem
- Straightforward power computations and thresholding after FFT
- Thresholding translates to rejecting a fragment
  - Potentially saves memory bandwidth
  - Use occlusion queries to determine if read-back is unnecessary

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Questions, Comments, Feedback?

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