Discretized Geometric Computations using GPUs

Dinesh Manocha
UNC-Chapel Hill
http://gamma.cs.unc.edu/

Arrangement Problems

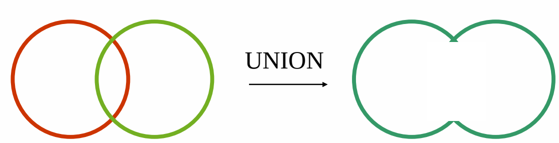
- **Arrangements**
  - Decomposition of space into connected open cells
  - Fundamental problem in computational geometry and related areas
- **Underlying structure in many geometric applications**
  - Swept Volumes
  - Minkowski Sums
  - CSG or Boolean operations
  - Many more…….

Basic Computational Pipeline

- Enumerate a set S of primitives that contribute to the final surface
- Compute the arrangement A(S) by performing intersection and trimming computations
- Traverse the arrangement and extract a substructure δA(S)

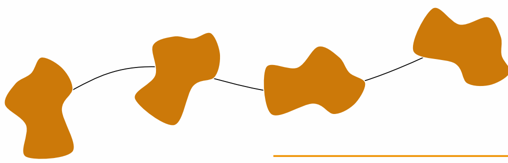
Example: CSG Union Operation

Boundary = outer envelope in the arrangement of the primitives



Swept Volume (SV)

- Volume generated by sweeping an object in space along a trajectory
- **Goal:** Compute a boundary representation of SV

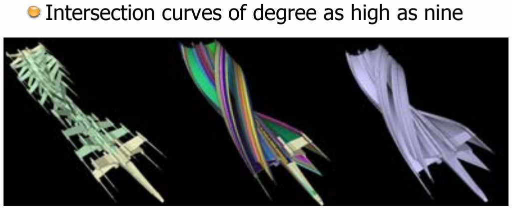


Swept Volume Computation

- Enumerate ruled and developable surfaces
- Boundary of SV = outer envelope of the arrangement

Swept Volume Computation

- X-Wing Model
  - 2496 triangles
  - 3931 ruled and developable surfaces
  - Intersection curves of degree as high as nine



Sweep Trajectory   Arrangement   Boundary of SV

Complexity of Arrangements

- High computational and combinatorial complexity
  - Super-quadratic in number of surfaces
- Accuracy and robustness problems
- No good practical implementations are available

Approximation Pipeline

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- Enumerate surface primitives
- Compute distance fields on a voxel grid
- Perform filtering operations on distance fields
- Use reconstruction algorithms to compute the boundaries of the arrangement

Approximation Pipeline

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- Enumerate surface primitives

Use GPUs:

- Compute distance fields on a voxel grid
- Perform filtering operations on distance fields

- Possibly use reconstruction algorithms to compute the boundaries

Talk Organization

SIGGRAPH 2003

- Fast distance field computation
- Applications

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Distance Fields

- **Distance Function**
  For a site a scalar function $f:R^n$ -> R representing the distance from a point P ε R^n to the site
- **Distance Field**
  For a set of sites, the minima of all distance functions representing the distance from a point P ε R^n to closest site

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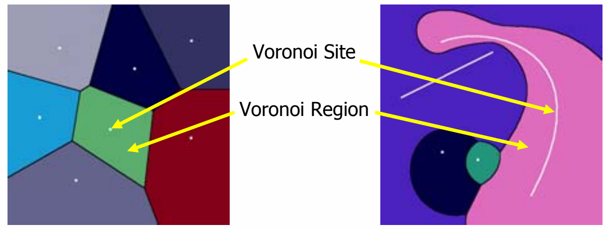
Voronoi Diagrams

Given a collection of geometric primitives, it is a subdivision of space into cells such that all points in a cell are *closer* to one primitive than to any other



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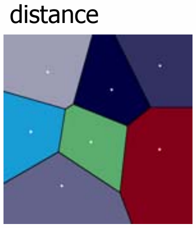
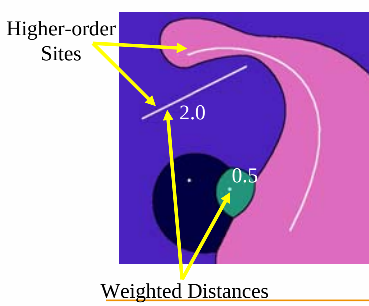
Ordinary Generalized

Ordinary
- Point sites
- Nearest Euclidean distance



Generalized
- Higher-order site geometry
- Varying distance metrics



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Voronoi Diagram & Distance Fields

- Minimization diagram of distance functions generates a Voronoi Diagram
- ***Projection of lower envelope*** of distance functions

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Distance Fields: Applications

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Collision Detection
Surface Reconstruction
Robot Motion Planning
Non-Photorealistic Rendering
Surface Simplification
Mesh Generation
Shape Analysis

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GPU Based Computation

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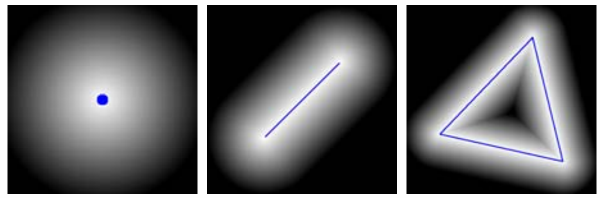
- **HAVOC2D, HAVOC3D [Hoff et al. 99,01]**
  - Evaluate distance at each pixel for all sites
  - Evaluate the distance function using graphics hardware



Point Line Triangle

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Approximating the Distance Function

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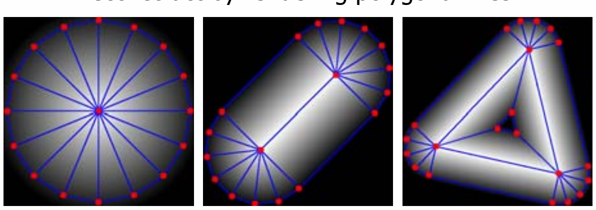
- Avoid per-pixel distance evaluation
- Point-sample the distance function
- Reconstruct by rendering polygonal mesh



Point Line Triangle

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GPU Based Computation

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- Triangular mesh approximation of distance functions
- Render distance meshes using graphics hardware
- Readback the frame buffer

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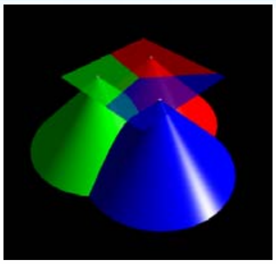
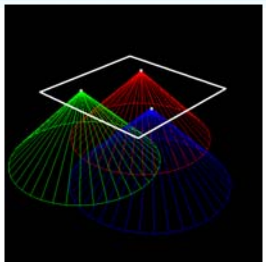
Meshing the Distance Function



Shape of distance function for
a 2D point is a cone



Need a bounded-error
tessellation of the cone

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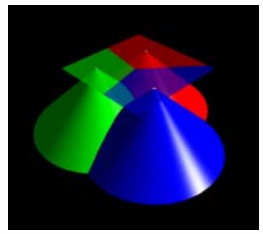
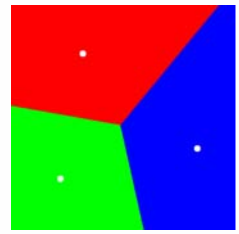
Graphics Hardware Acceleration

- Rasterization to reconstruct distance values
- Depth test to perform minimum operator



Perspective, 3/4 view



Parallel, top view

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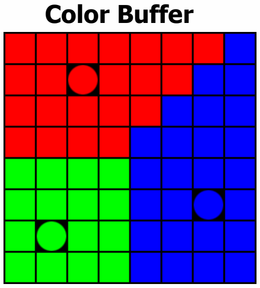
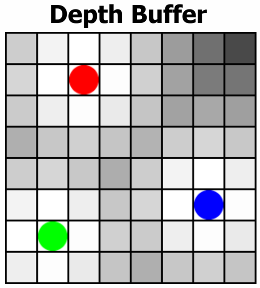
Results in the Frame Buffer

Color Buffer



Voronoi Regions

Depth Buffer



Distance Field

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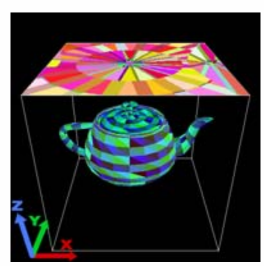
3D Voronoi Diagrams

- Graphics hardware can generate one 2D slice at a time
- Sweep along 3rd dimension (Z-axis) computing 1 slice at a time



Distance Field of the Teapot Model

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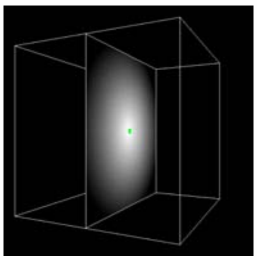
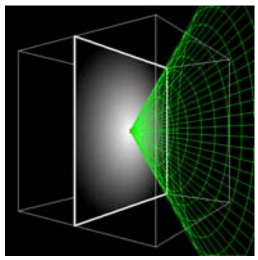
Shape of 3D Distance Functions

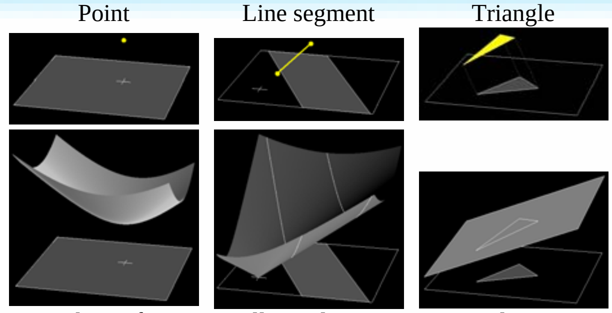


Slices of the distance function for a 3D point site



Distance meshes used to approximate slices

Shape of 3D Distance Functions



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Bottlenecks in HAVOC3D

- **Rasterization:**
  - Distance mesh can fill entire slice
  - Complexity for *n* sites and *k* slices = O(*kn*)
  - Lot of Fill !
- **Readback:**
  - Stalls the graphics pipeline
  - Not suitable for interactive applications

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Improved Distance Field Computation (DiFi)

- Overview
- Culling techniques
- Application
- Implementation and results

[Sud and Manocha 2003]

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Distance Field Applications

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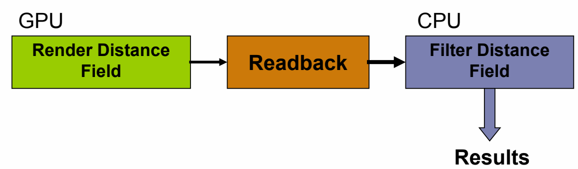
Application Framework



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Distance Field Applications

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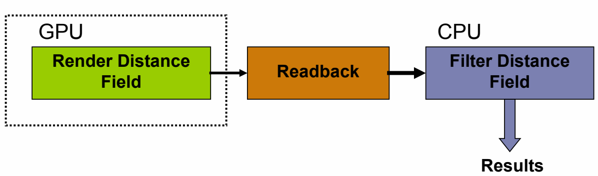
Reduce fill: Cull using estimated voronoi region bounds



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Improved Distance Field Computation (DiFi)

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Reduce fill: Cull using estimated voronoi region bounds

- **Along Z:** Cull sites whose voronoi regions don't intersect with current slice
- **In XY plane:** Restrict fill per site using planar bounds of the voronoi region

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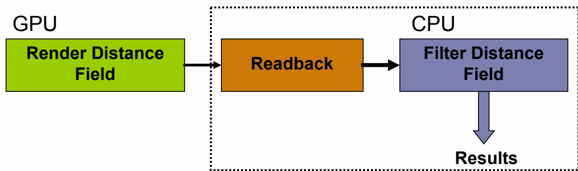
Improved Distance Field Computation (DiFi)

Avoid readbacks: Perform distance field application on GPU



Voronoi Diagram Properties

- Within a bounded region, all voronoi regions have a bounded volume

9 Sites, 2D

Voronoi Diagram Properties

- Within a bounded region, all voronoi regions have a bounded volume
- As site density increases, average spatial bounds decrease

27 Sites, 2D

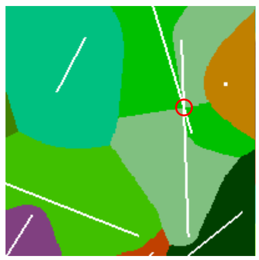
Voronoi Diagram Properties

Voronoi regions are connected

- Valid for l_2 , l_{inf} etc. norms

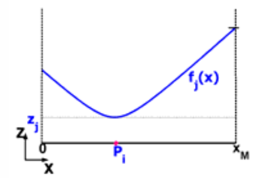
Voronoi Diagram Properties

- **Voronoi regions are connected**
  - Valid for l_2 , l_{inf} norms
- **Special cases:** Overlapping features



Voronoi Diagram Properties

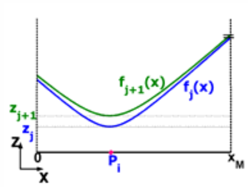
- High distance field coherence between adjacent slices
- Change in distance function between adjacent slices is bounded



Distance functions for a point site P_i to slice Z_j

Voronoi Diagram Properties

- High distance field coherence between adjacent slices
- Change in distance function between adjacent slices is bounded



Distance functions for a point site P_i to slice Z_{j+1}

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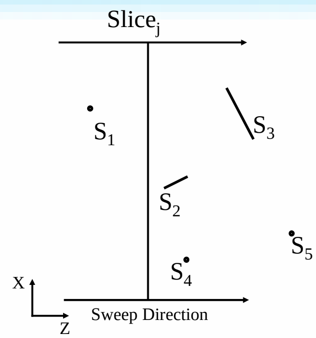
Site Culling: Classification

For each slice partition the set of sites



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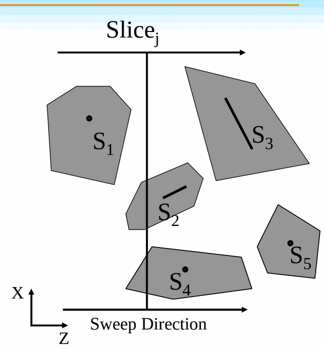
Site Culling: Classification

For each slice partition the set of sites using voronoi region bounds



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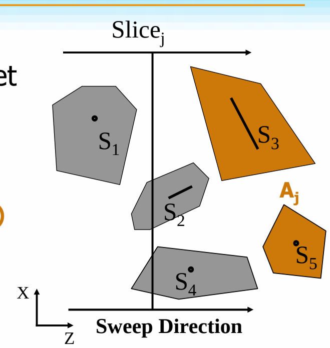
Site Culling: Classification

For each slice partition the set of sites, using voronoi region bounds:

Approaching (A_j)



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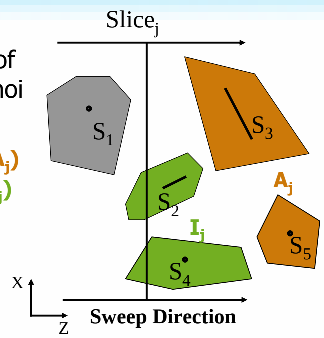
Site Culling: Classification

For each slice partition the set of sites, using voronoi region bounds:

- **Approaching (A_j)**
- **Intersecting (I_j)**



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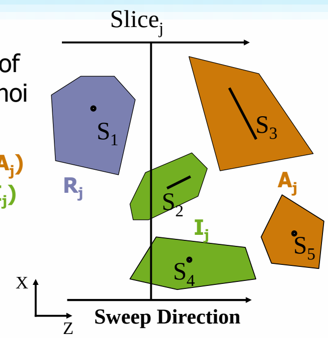
Site Culling: Classification

For each slice partition the set of sites, using voronoi region bounds:

- **Approaching (A_j)**
- **Intersecting (I_j)**
- **Receding (R_j)**



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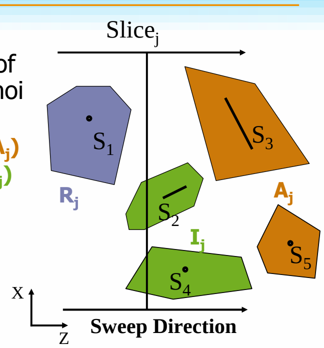
Site Culling: Classification

- For each slice partition the set of sites, using voronoi region bounds:
  - **Approaching (A_j)**
  - **Intersecting (I_j)**
  - **Receding (R_j)**
- Render distance functions for Intersecting sites only



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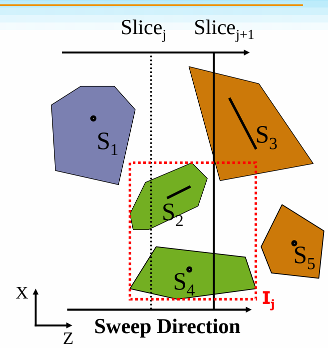
Coherence: Adjacent Slices

Updating $\mathbf{I_j}$

$\mathbf{I_{j+1}} = \mathbf{I_j} \ldots$

Previously intersecting



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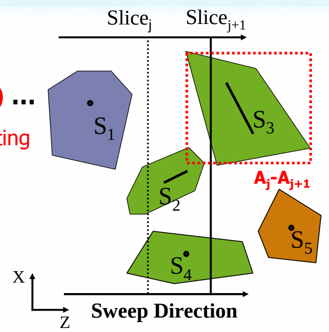
Coherence: Adjacent Slices

- **Updating I_j**

$I_{j+1} = I_j$
$+ (A_j - A_{j+1})$ …

Approaching → Intersecting



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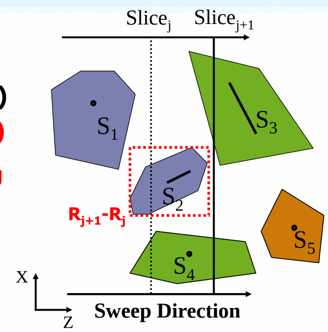
Coherence

- **Updating I_j**

$I_{j+1} = I_j$
$+ (A_j - A_{j+1})$
$- (R_{j+1} - R_j)$

Intersecting → Receding



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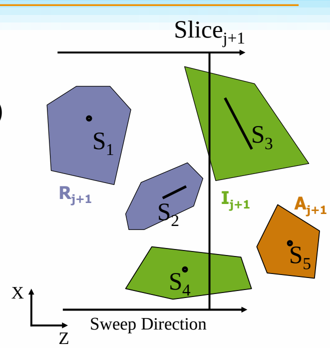
Coherence

- **Updating I_j**

$I_{j+1} = I_j$
$+ (A_j - A_{j+1})$
$- (R_{j+1} - R_j)$



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Estimate Potentially Intersecting Set (PIS)

- Computing exact intersection set = Exact voronoi computation
- **Conservative Solution**:
  - Use hardware based occlusion queries
  - Determine number of visible fragments
  - Computes *potentially intersecting* sites (PIS) $\hat{I}$

$$\hat{I}_j \supseteq I_j$$

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Improved Distance Field Computation (DiFi)

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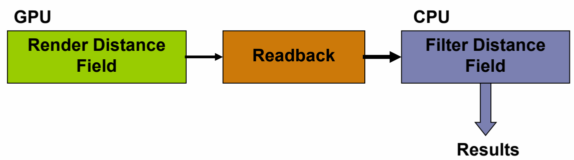
Application: GPU Computation

Avoid frame buffer readbacks (e.g. HAVOC3D)



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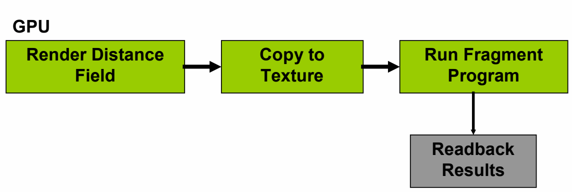
Application: GPU Computation

- Avoid frame buffer readbacks
- Distance field is a 3D grid: SIMD applications suitable for GPU computation



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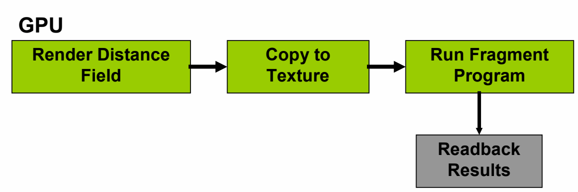
Application: GPU Computation

- Size of Results << Size of Distance Field
- Readback cost of results << Readback of Distance Field



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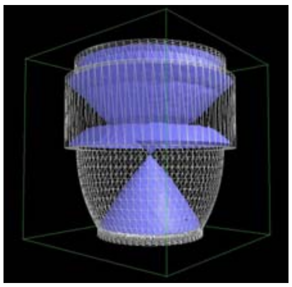
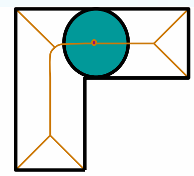
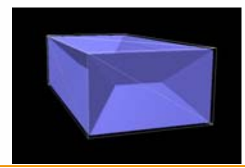
Application to Medial Axis Computation



Blum Medial Axis

- Locus of centers of maximal contained balls
- Well-understood medial representation
- **Applications**
  - Shape analysis
  - Mesh generation
  - Motion planning
- Exact computation is hard





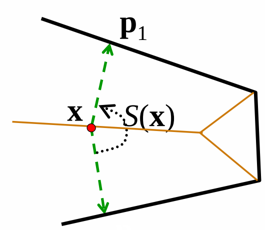
θ-Simplified Medial Axis M_θ

- A subset of the full medial axis *M*
- Relies on *separation angle* from points on the medial axis to the boundary
- More stable than Blum medial axis

[Foskey, Lin and Manocha 2002]

Separation Angle

- Angle separating the vectors from x to nearest neighbors
- If more than 2 nearest neighbors, maximum angle is used



Large Separation Angle

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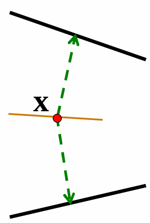
Point is roughly *between* its nearest neighbor points



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Small Separation Angle

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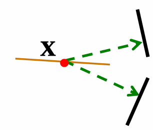
Point is *off to one side* of its nearest neighbor points



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Simplified Medial Axis

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$$M_\theta = \{\mathbf{x} \in M \mid S(\mathbf{x}) > \theta\}$$

- Start with medial axis M
- Eliminate portions with $S(x) \leq \theta$

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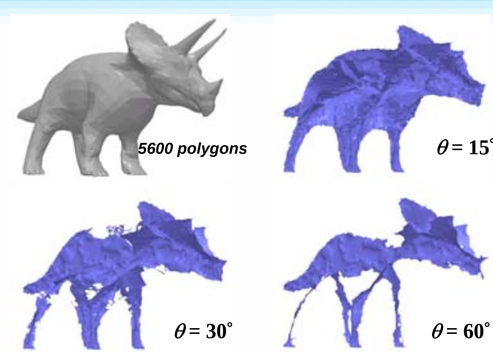
3D Example: Triceratops

SIGGRAPH 2003



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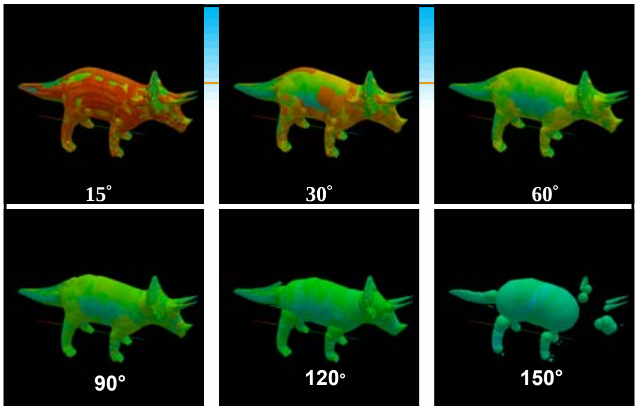

Shape Simplification using Simplified Medial Axis

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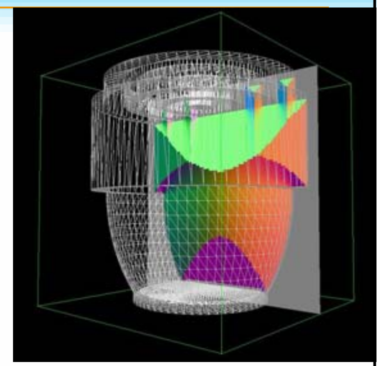
Direction Field

- Gradient of Distance Field
- Direction image rendered for each slice (constant z)
- Direction vectors encoded as RGB triples
- Length encoded in depth buffer



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Improved Distance Field Computation (DiFi)

- Overview
- Culling techniques
- Applications
- Implementation and Results

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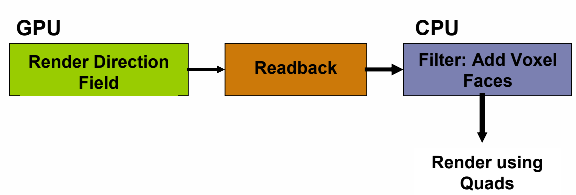
Simplified MAT Computation with GPU/CPU

Computation using **HAVOC3D** [Foskey et al. 2002]



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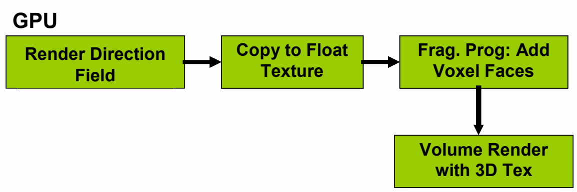
Simplified MAT Computation using GPUs

Computation using **DiFi** [Sud et al. 2003]



Implementation

Dell Workstation with

- Pentium4 CPU at 2.8Ghz
- NVidia GeForce FX5800 Ultra GPU
- 2GB RAM
- Windows 2000

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Results: MAT Computation

Total time =

Time(Direction Field) + Time(Filter Voxels)

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Direction Field Computation

- **4 -20 times speedup over HAVOC3D**

Model	Polys	Resolution	HAVOC (s)	DiFi (s)
Shell Charge	4460	128x126x126	31.69	3.38
Head	21764	79x106x128	52.47	13.60
Bunny	69451	128x126x100	212.71	36.21
Cassini	90879	94x128x96	1102.01	47.90

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Filter Voxels

- 2 - 75 times speedup

Model	Resolution	CPU (s)	GPU (s)
Shell Charge	128x126x126	3.50	0.14
Head	79x106x128	0.18	0.08
Bunny	128x126x100	0.68	0.13
Cassini	94x128x96	7.59	0.1

Filter Voxels: CPU vs. GPU

- Depends on grid size
- 2 - 75 times speedup via GPUs

Model	Resolution	T(CPU) (s)	T(GPU) (s)
Shell Charge	128x126x126	3.50	0.14
Head	79x106x128	0.18	0.08
Bunny	128x126x100	0.68	0.13
Cassini	94x128x96	7.59	0.1

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Improved Distance Field Computation (DiFi)

- Overview
- Culling Techniques
- Applications
- Results
- Live demo

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Demo: MAT computation

- Triceratops model
- Shell model : designed using a CAD system
  - Sharp edges
  - Non-manifold
- Dell Mobile Workstation with NVIDIA QuadroFX GoGL Card (128 MB Video Memory)

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Other Applications

- Max-norm computation
- Swept volumes
- CSG operations on complex models

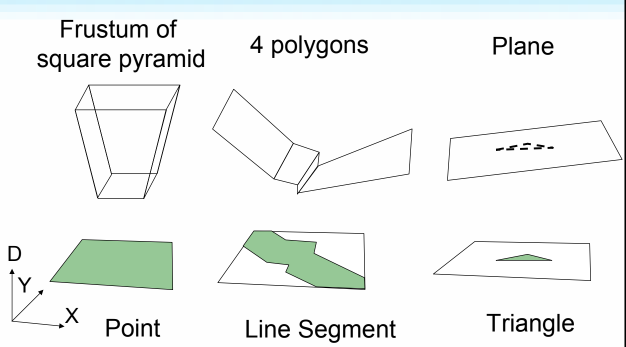
Max-Norm (l_∞) Computation

- Max-Norm
  - Natural metric for axis-aligned voxels

$$\| \mathbf{p} \|_\infty = \max(|x|, |y|, |z|)$$

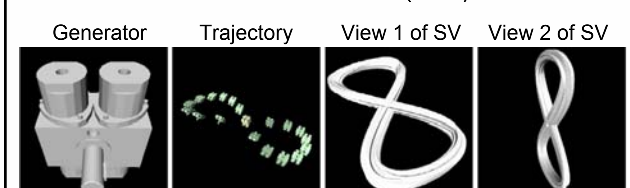
- Useful for voxelization, error deviation, etc.

Linear Distance Functions for l_∞ Computations



Swept Volume Computation

Used Distance Fields (DiFi)



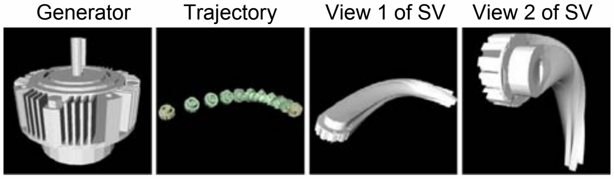
2,280 triangles | 1,152 surfaces | Time = 12 secs

[Kim et al. 2003]
http://gamma.cs.unc.edu/SV

Results: Swept Volume

Input Clutch Model

Generator | Trajectory | View 1 of SV | View 2 of SV

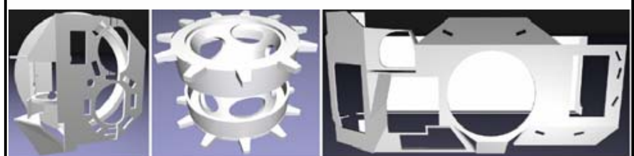


2,116 triangles | 1,175 surfaces | Time = 21 secs

[Kim et al. 2003]
http://gamma.cs.unc.edu/SV

Boundary Evaluation of Complex CSG Models

30-40 solids defined using 2-7 Boolean operations
8-13 secs per solid



Turret | Drivewheel | Hull

[Varadhan et al. 2003]
http://gamma.cs.unc.edu/recons

Boundary Evaluation of Complex CSG Models

Bradley Fighting Vehicle
1200 solids
8,000 CSG operations
Took 2 hours

[Varadhan et al. 2003]
http://gamma.cs.unc.edu/recons

References

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- M. Foskey, M. Lin, and D. Manocha. "Efficient computation of a simplified medial axis". *Proc. of ACM Solid Modeling*, 2003.
- Y. Kim, G. Varadhan, M. Lin and D. Manocha. "Fast approximation of swept volumes of complex models". *Proc. of ACM Solid Modeling, 2003.*
- G. Varadhan, S. Krishnan, Y. Kim and D. Manocha. "Feature-based Subdivision and Reconstruction using Distance Field". *Proc. Of IEEE Visualization, 2003 (to appear).*

Conclusions

- Discretized geometric computations using GPUs
- Arrangement and envelope comptuations
- Fast distance field computation
- Application to medial, swept volume and CSG computations
- Benefits
  - Improved performance
  - Robust implementations

The End

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