Motion Planning & Physically-based Modeling Using GPUs

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http://gamma.cs.unc.edu/PD
http://gamma.cs.unc.edu/CULLIDE
http://gamma.cs.unc.edu/ICE

Organization

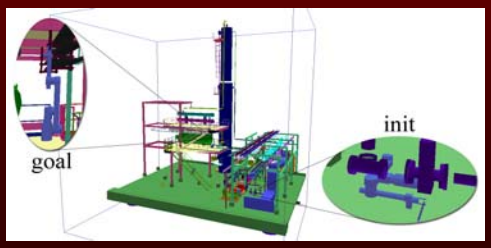
- Motion Planning
- Proximity Queries
- Phyiscally-based Simulation

Organization

- **Motion Planning**
- Proximity Queries
- Penetration Depth Estimation

Motion Planning

Given the initial and goal configurations, the geometric description of the robot/agent and the environment, find a viable path if one exists.



Applications

- Assembly Planning
- Car Painting
- Waste Cleaning

- Character animation
- Virtual human or artificial life
- Molecular docking
- Surgical Planning
- Virtual prototyping
- Pipe layout

Two Classes of Planning Algorithms

- Criticality-based
  - Complete
  - Computational expensive
  - Difficult to implement
- Random Sampling
  - Probablistically complete
  - Simple to implement
  - Fast in practice for most scenarios
  - Performance degrading on narrow passages

GVD: Maximally Clear Points

Points with the largest distance values to nearby obstacles

Planning Based on GVD & MAT

- O'Dunlaing, Sharir and Yap [1983]
- Canny and Donald [1987]
- Latombe [1991]
- Choset, Burdick, et al. [1994-1999]
- Vleugels and Overmars [1996,1997]
- Guibas, Holleman and Kavraki [1999]
- Wilmarth, Amato and Stiller [1999]

…… and others ……

Basic Ideas

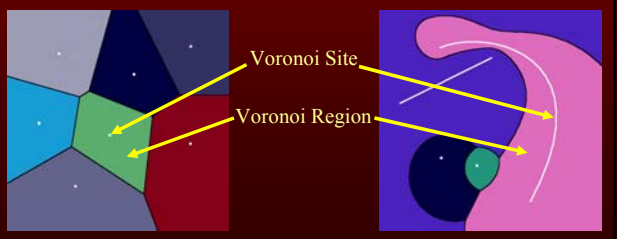
- Use of rasterization graphics hardware for real-time motion planning in dynamic environments
- Discrete GVD of any primitive types and approximate distance function with bounded errors
- Simple and "implementation-friendly" acceleration techniques for Voronoi-based robot motion planning
  - Construct Voronoi roadmap
  - Biasing path selection
  - Quick collision rejection test
  - Sampling near medial axis
  - Reduce search space
  - Establish milestones

Outline

- **Review: Computing GVD with Graphics Hardware**
- Motion Planning with GVD
- Implementation & Results
- Summary

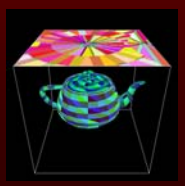
Voronoi Diagram

Given a collection of geometric primitives, it is a subdivision of space into cells such that all points in a cell are *closer* to one primitive than to any other



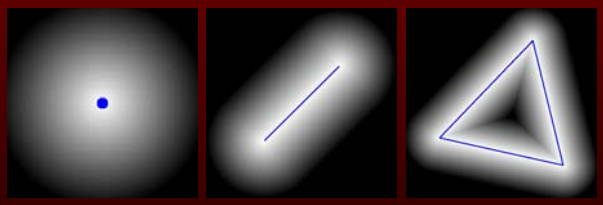
Accelerated Computation of GVD [Hoff et al. SIGGRAPH'99]

- Compute a discrete Voronoi diagram by rendering a three dimensional distance mesh for each Voronoi site.
- The polygonal mesh is a bounded-error approximation of a distance functions btw a site and a 2D planar grid of points.
- Each site is assigned a unique color & distance mesh rendered in that color.
- For each sample point, the graphics system computes the closest site and the distance to that site using polygon scan-conversion and the Z-buffer depth comparison.



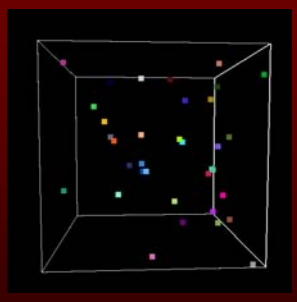
The Distance Functions for 2D Primitives

Evaluate distance at each pixel for all sites
Accelerate using graphics hardware



Point Line Triangle

3D Voronoi Diagrams

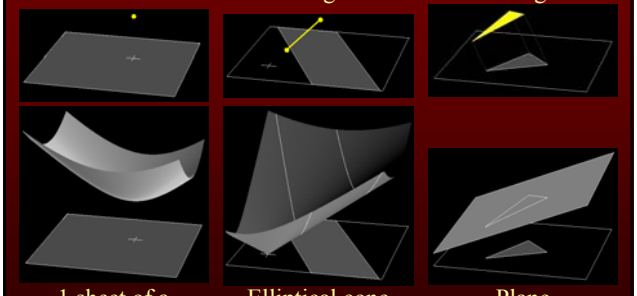


Point sites

Graphics hardware can generate one 2D slice at a time

3D Distance Functions

Point Line segment Triangle



1 sheet of a hyperboloid Elliptical cone Plane

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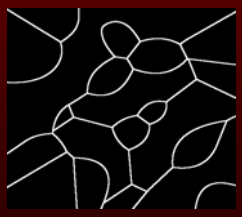
Basic Approaches

- Depth Buffer – providing distance function and distance gradient (finite difference)
- Color Buffer – building Voronoi graphs
- Combination of Both
  - Compute weighted Voronoi graphs
  - Voronoi vertices used for milestones
  - Weighted edges used for selecting paths
  - Distance values for quick rejection

Voronoi Boundary

- For each voxel in discrete GVD, associate a color corresponding to an object ID and a distance value to this obstacle.
- To extract the boundary, use continuation method similar to iso-surface extraction -- starting from a seed point and step to next by bracketing boundary curves in a 2x2x2 region of sampled points.



Constructing Voronoi Roadmap

- Identify Voronoi vertices
- Extract Voronoi boundary
- Build Voronoi graph
- Select path based on edge weights
- Incrementally construct Voronoi roadmap
  - local planner (PFF) between milestones
  - quick collision rejection test & exact CD

Voronoi Roadmap/Graph for Dynamic Environments

Outline

- Computing GVD with Graphics Hardware
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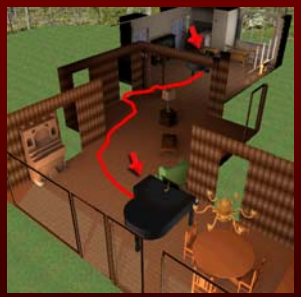
Planning Using Voronoi Diagrams

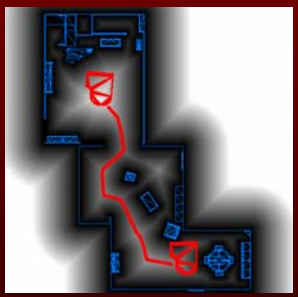
- Use the Voronoi graphs to find the near optimal path (in terms of both path length and clearance to nearby ostacles)
- The graphs are updated on the fly using GPU
- Use the potential field approach to orient the robot
- Applicable to both static and dynamic environments

[Hoff, et al, ICRA'00]

Real-time Motion Planning : Static Scene



Plan motion of piano (arrow) through 100K triangle model



Distance buffer of floorplan used as potential field

Real-time Motion Planning : Dynamic Scene

Plan motion of music stand around moving furniture

Distance buffer of floor-plan used as potential field

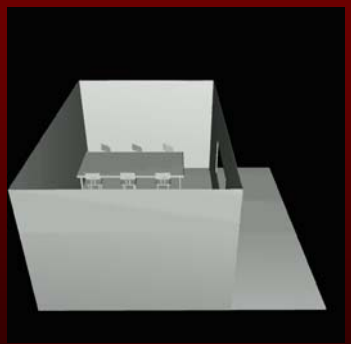
Voronoi-Based Sampling

- Use hardware accelerated computation of generalized Voronoi diagram (GVD) for PRM with Voronoi-based sampling
- Classify narrow passages to select different sampling strategies
- Quick collision rejection tests using hardware computed distance functions
- 25 to 1000% speedup over uniform sampling on preliminary benchmarks

[Pisula, **et al, WAFR'00**]

3D Benchmark (I)

3D Benchmark (II)



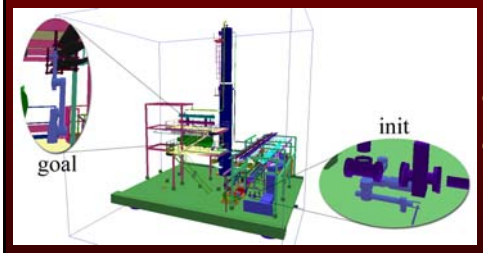
Model Courtesy of J-P Laumond at LAAS, Toulouse

Voronoi-Based Hybrid Planner

- Use Voronoi Graphs as the estimated path
- Use the curvature of the path to orient the robot
- When a collision occur, use random sampling to correct the estimated path

[Foskey et al, IROS'01]

Articulated Robots: Crane Complex



GVG: 138.7s

Query: 334.7s

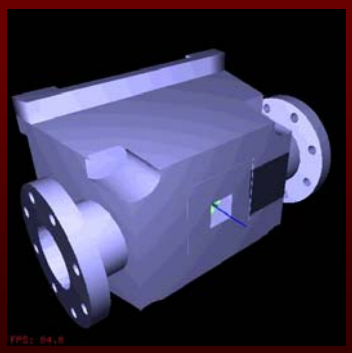
Model courtesy Jean-Paul Laumond

Constraint-Based Motion Planning

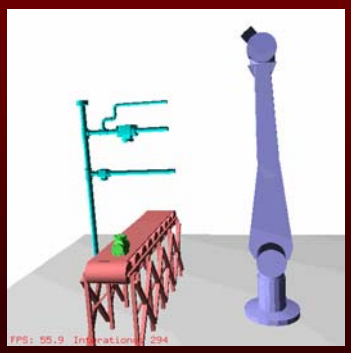
- Transform a motion planning problem into a constraint-based dynamic simulation
- Use hardware accelerated computation of distance fields for potential functions and quick collision rejection
- Applicable to both dynamic and static environments with moving obstacles and multiple collaborative agents

[Garber & Lin'02]

Collaborating Agents



Virtual Protoyping



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Summary

- Techniques to exploit graphics hardware for ***real-time*** motion planning of a rigid robot in 3D
- Simple and easy to implement
- Applicable to both static and dynamic environments
- Extended to Voronoi-based sampling for PRM
- Core of the Voronoi-based hybrid-planner
- Central to Constrained Motion Planning

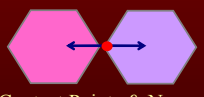
Organization

- Motion Planning
- **Proximity Queries**
- Physically-based Simulation

Proximity Queries

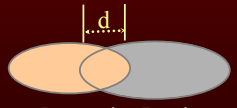
*Geometric reasoning of spatial relationships among objects (*in a dynamic environment)

Collision Detection



Contact Points & Normals

Closest Points & Separation Distance

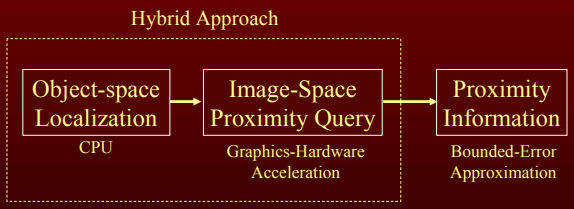


Penetration Depth

Applications

- Dynamic simulation – ***contact force calculation***
- Haptic Rendering -- ***restoring force computation***
- Computer Animation – ***motion control***
- Motion Planning – ***distance computation***
- Rapid Prototyping – ***tolerance verification***
- Virtual Environments -- ***interactive manipulation***
- Simulation-Based Design – ***interference detection***
- Engineering analysis – ***testing & validation***
- Medical Training – ***contact analysis and handling***
- Education – ***simulating physics & mechanics***

Our Approach

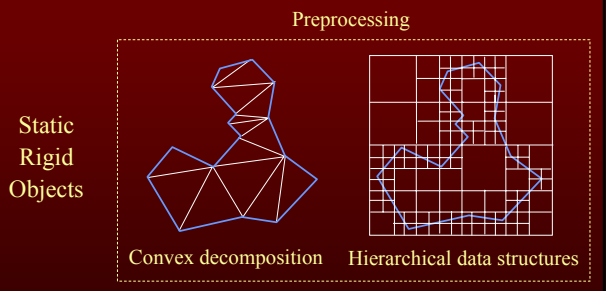


[Hoff, Zaferakis, Lin & Manocha, I3D01]

Related Work

- Proximity query algorithms
  - Exact, object-space
  - Approximate, image-space
- Distance fields

Exact, Object-Space Algorithms



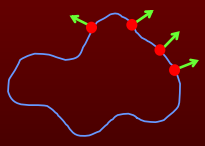
Gilbert88 , Lin90, Cameron97, Ehmann00

Gottschalk96, Hubbard93, Johnson98, Quinlan94

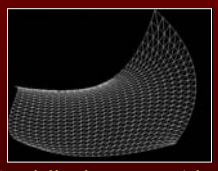
Exact, Object-Space Algorithms

Restricted Cases

Dynamic Deformable Objects



Known surface motion trajectories



Specialized geometry (cloth)

Snyder93

Baraff92, Baraff98

Approximate, Image-Space Algorithms

Algorithms simplified and accelerated using graphics hardware:

- Visibility: Sutherland74, Catmull75
- Intersections: Rossignac92, Myskowski95, Baciu98
- CSG: Goldfeather86, Rossignac90, Stewart00
- Robot motion-planning: Lengyel90, Hoff00
- Voronoi diagrams: Hoff99

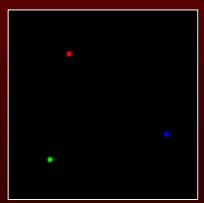
Advantages:

- Simplicity
- Bounded error
- Linear complexity
- Robustness

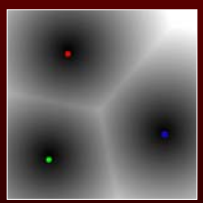
Distance Fields

Distance fields give distance to nearest object from any point:

- Fast marching method Sethian96
- Generalized Voronoi diagrams Hoff99
- Adaptively-sampled distance fields Frisken00
- Distance fields for penetration computation Fisher01



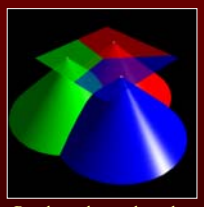
3 point objects



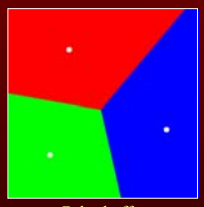
Distance field

Distance Fields

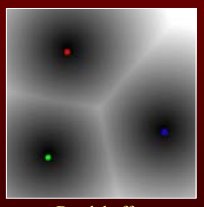
Voronoi diagram computation using graphics hardware (Hoff99)



Render polygonal mesh approximations of object distance fields



Color buffer



Depth buffer

Result after compositing distance fields using minimum depth test

Our Goal

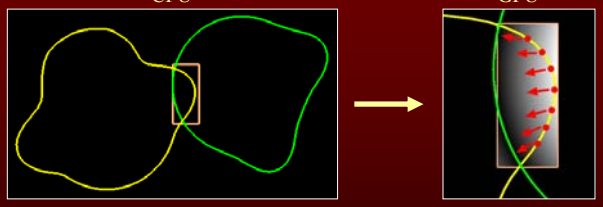
Proximity query algorithm with the following properties:

- Performs all proximity queries
- Handles non-convex primitives
- Requires no precomputation or complex data structures
- Fast and efficient
- Robust
- Portable
- Bounded error

No previous algorithm found, even in 2D!

Our Hybrid Approach

CPU GPU



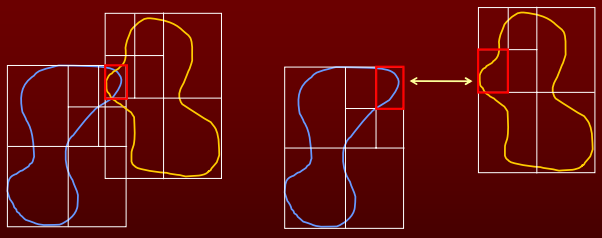
Coarse object-space geometric localization

Image-space proximity queries

Balance load by varying localization coarseness and error bound

[Hoff, Zaferakis, Lin & Manocha, I3D01]

Geometric Localization



Potential Intersections Potential Closest Points

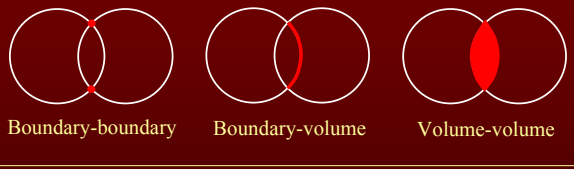
- Reduces the number of pixels processed per image-space query
- Vary max depth in hierarchy to balance CPU/graphics loads

Image-Space Proximity Queries

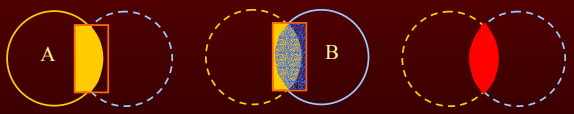
- Intersections
- Distance field and gradients

- Contact points and normals
- Closest points
- Separation distance and direction
- Penetration depth and direction

Intersections

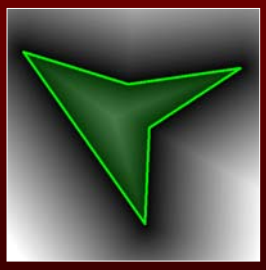


Boundary-boundary Boundary-volume Volume-volume

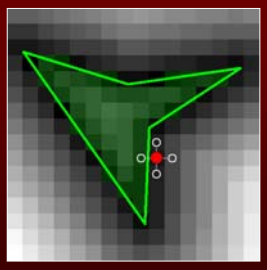


Draw A → Draw B → Intersection = Overwritten Pixels

Distance Field and Gradients



Draw polygon to encode negative sign in a buffer



Central difference to compute gradient at a pixel

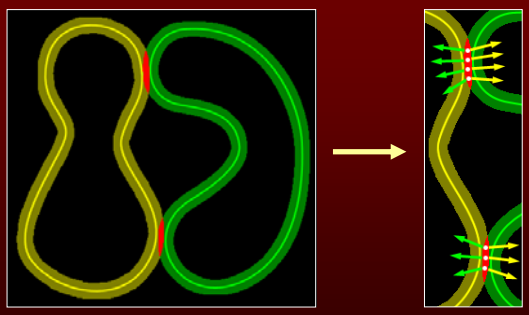
Image-Space Proximity Queries

- Intersections
- Distance field and gradients

↓

- Contact points and normals
- Closest points
- Separation distance and direction
- Penetration depth and direction

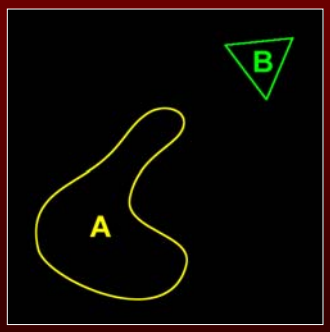
Contact Points and Normals



Close contact points as intersection of enlarged boundary

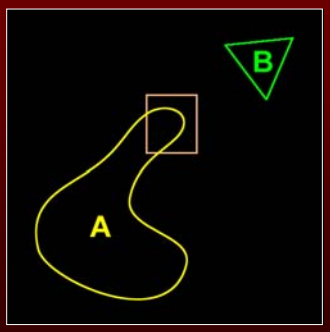
Normals from gradient of distance field

Closest Points



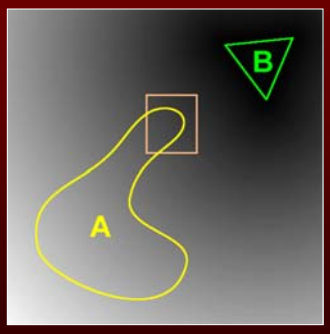
Find closest point on object A to object B

Closest Points



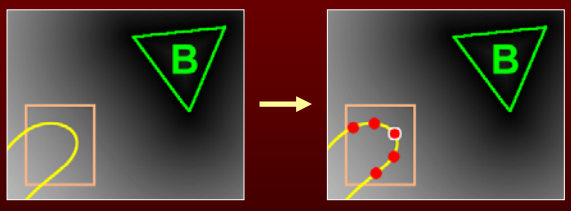
Localize region on A containing the closest point

Closest Points



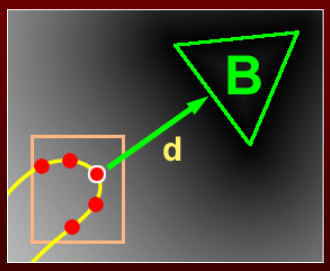
Compute distance field of B in A's localized region

Closest Points



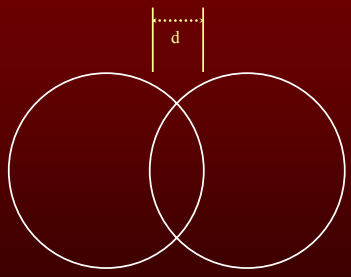
- Render boundary of A to find set of potential closest points
- Check distances at each point to find the closest

Separation Distance and Direction



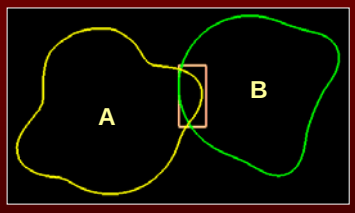
Distance at closest point = min separation distance
Gradient at closest point = separating axis direction

Penetration Depth

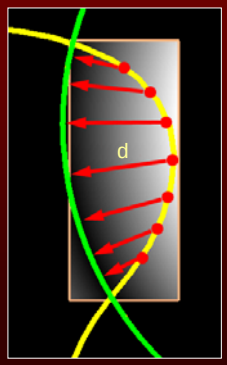


Minimum translational distance needed to separate objects

Penetration Depth and Direction for a Point

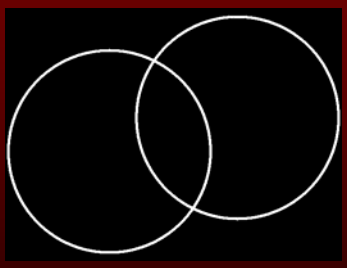


Compute penetration depth and direction for A's boundary points that intersect B's interior

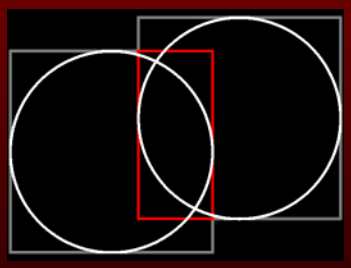


Putting it all together…

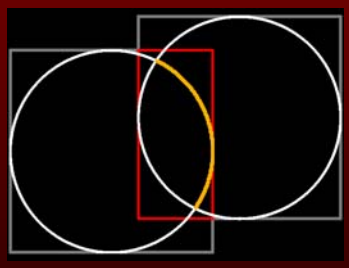
Collision response between 2 deformable objects



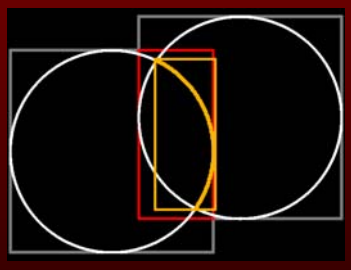
Localize Potential Intersections



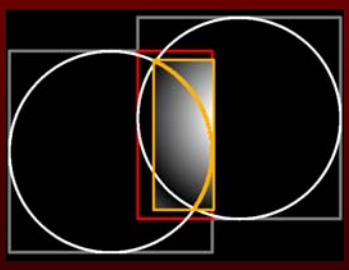
Find Penetrating Points



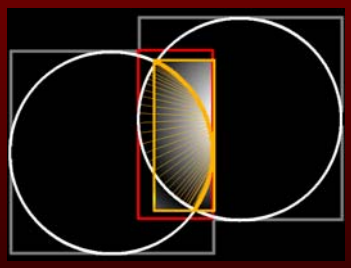
Tighten Localized Region



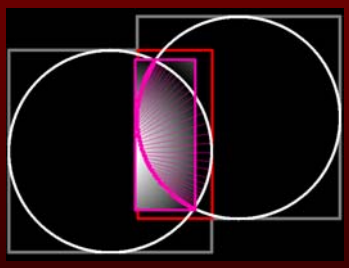
Compute Distance Field



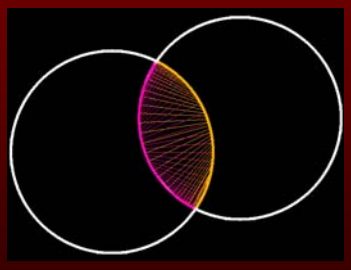
Compute Penetration Depth and Direction



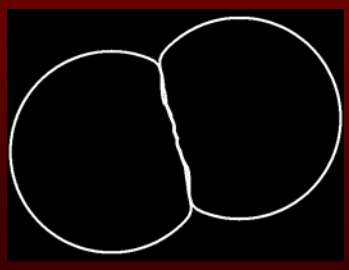
Repeat Process for Other Object



Use Proximity Info for Collision Response



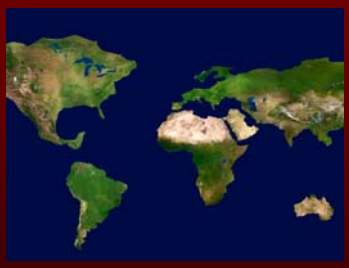
Separation Forces from Penetration Depth



Demonstrations

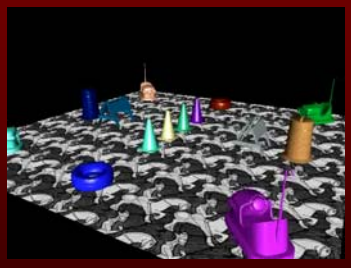
- Simulation test cases:
  - Rigid and deformable objects
  - Different contact scenarios
- Collision response in 2 simulation strategies:
  - Non-penetration constraint, backtracking
  - Unconstrained, penalty-based
- In each demo:
  - No precomputation
  - Interactive frame rates
  - Bounding-box intersection localization
  - Pentium III 800, nVidia GeForce256

MAP



- Non-convex, rigid objects
- Frequent simultaneous close contacts
- Unconstrained, penalty-based

BUMPER CARS

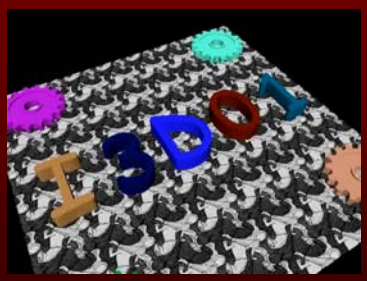


- Convex, rigid objects
- Less frequent contact
- Non-penetration constraint, backtracking

LINKS

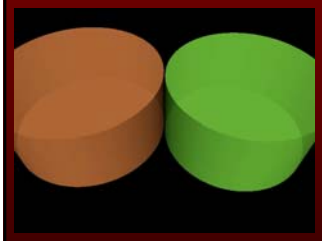
- Non-convex, rigid objects
- Frequent interlocking contacts
- Unconstrained, penalty-based

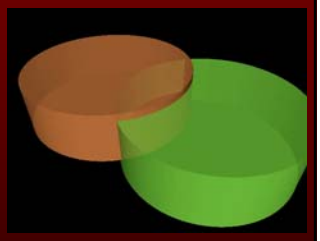
GEARS



- Non-convex, rigid objects
- Less frequent interlocking contacts
- Unconstrained, penalty-based

WAVY





- Non-convex, deformable objects
- Continuous contact
- Unconstrained, penalty-based

Resolving extreme penetration

Performance

Average Total Per-frame Proximity Query Times					
Demo	**Objects**	**Lines**	**GeForce2**	**InfiniteReality2**	**ATI Rage Pro LT**
Map	6	719	0.281ms	0.901ms	0.434ms
Gears	13	391	0.015	0.026	0.064
Links	15	440	0.020	0.052	0.038
Cars	18	266	0.007	0.026	0.015
Wavy	2	200	1.030	2.360	2.990

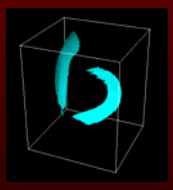
Performance timings for dynamics simulations. The number of objects, number of line segments, and the average total time in milliseconds to run proximity queries on all objects in the scene per frame is reported. Timing data was gathered from three machines: a Pentium-III 933MHz desktop with a 64Mb GeForce2, a SGI 300MHz R12000 with InfiniteReality2 graphics, and a Pentium-III 750Mhz laptop with ATI Rage Pro LT graphics.

Effects of Changes in Error Tolerance

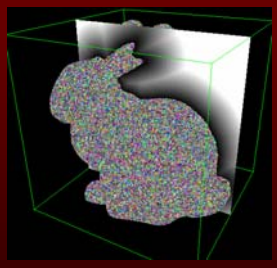
Effects of Error Tolerance on Performance of Wavy			
Error	**GeForce2**	**InfiniteReality2**	**ATI Rage Pro LT**
d/4	0.710ms	1.270ms	5.560ms
d/2	0.315	1.000	1.850
d	0.211	0.930	0.895
2d	0.176	0.879	0.631
4d	0.165	0.876	0.535

The effect on performance when changing the distance error tolerance *d*. We used proximity queries on the **wavy** demo with no collision response. The error determines the number of pixels used in the image-based operations. Systems with low graphics performance are more directly affected by the choice of *d* (see ATI Rage Pro LT); however, as the error is increased there is less dependence on graphics performance and the faster laptop CPU overtakes the InfiniteReality2 system.

Recent Results: Extension to 3D



3D intersections

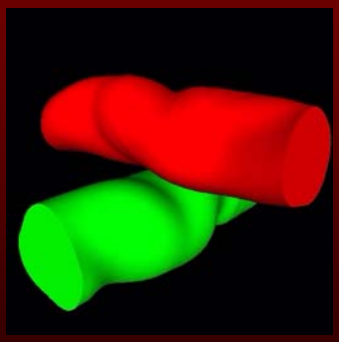


3D distance field

Real-time queries: Deformable models

- Non-convex, complex 3D deformable objects
- ~20000 polygons each cylinder
- Continuous contact
- Unconstrained, penalty-based
- Intersection region in blue
- Distance field gradients in red and green
- No pre-computation

[Hoff, Zaferakis, Lin, Manocha'02]



Resolving extreme penetration

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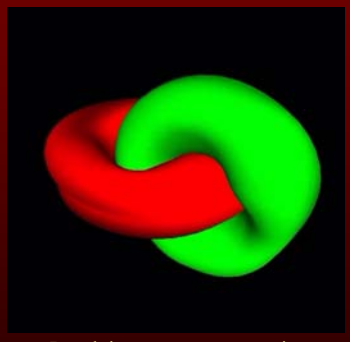
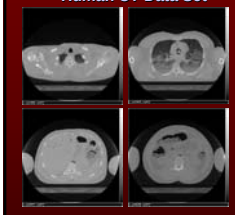


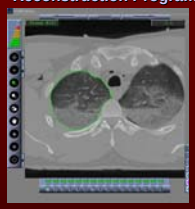
Resolving extreme penetration

NURBS Torso Models [Seeger et al.'00]

Sample Slices from the Visible Human CT Data Set



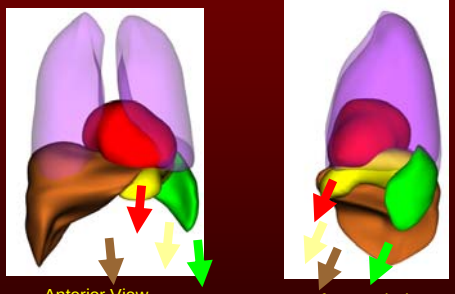
SURFdriver Surface Reconstruction Program



NCAT Phantom (anterior view)

3D NURBS Organ Models

4D NURBS Respiratory Model

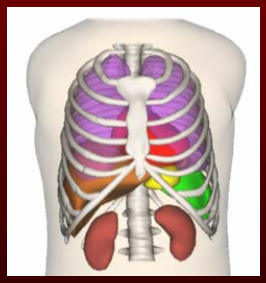


Anterior View

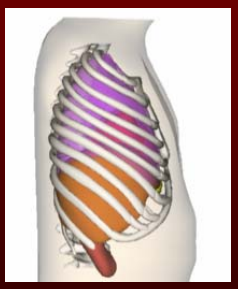
Left Lateral View

Motion Analysis of the Diaphragm, Heart, Liver, Spleen, and Stomach

Proximity Queries between Organs



Anterior View



Right Lateral View

[Seeger et al.'00]

Modeling the Heart Structures

Heart exterior

Contact computations between Inner chambers

Proximity Queries using Graphics H/W

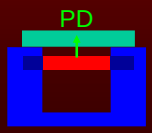
- Performs all proximity queries
- Handles general non-convex & deformable primitives
- Requires no pre-computation or complex hierarchical data structures
- Fast and efficient

Some Observations

- Appropriate for most of applications where numerical accuracy is not the key
- Running time related to the rendering time
- Error bound provides smooth dial between performance and level of approximation
- Varying the max depth in the hierarchical localization allows balancing load between CPU and graphics

Definition

- Penetration Depth (PD)
  - Minimum translational distance to separate two intersecting objects

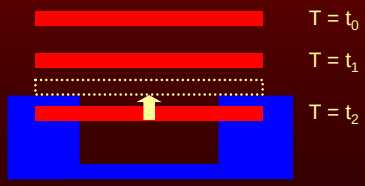


Motivation

- Contact Handling in Rigid Body Simulation
  - Penalty-based method for contact resolution
  - Time stepping in general simulation framework

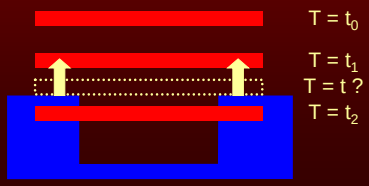
Motivation

- Penalty-based Method



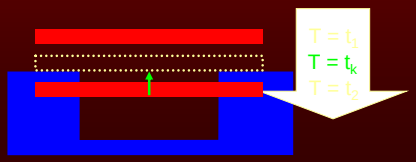
Motivation

- Time stepping method
  - Estimate the time of collision (TOC)



Motivation

- Time stepping method using PD
  1. Compute the penetration depth
  2. Estimate the TOC by interpolation in time domain



PD Applications

- Rigid body dynamic simulation
- Robot motion planning for autonomous agents and animated characters
- Haptic rendering
- Tolerance verification for CAD models

Previous Work

- Convex polytopes
  - [Cameron '86 '97], [Dobkin et al. '93], [Agarwal et al. '00], [Bergen '01], [Kim et al. '02]
- Local solutions for deformable models
  - [Susan and Lin '01], [Hoff et al. '01]
- No solutions existed for non-convex models

Overview

- Preliminaries
  - Minkowski sum-based Framework
- Basic PD Algorithm: A Hybrid Approach
- Acceleration Techniques
  - Object Space Culling
  - Hierarchical Refinement
  - Image Space Culling
- Application to Rigid Body Dynamic Simulation

Overview

- **Preliminaries**
  - **Minkowski sum-based Framework**
- Basic PD Algorithm: A Hybrid Approach
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  - Object Space Culling
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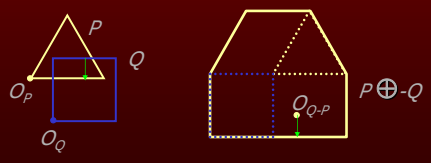
Preliminaries

- Local solutions might not have any relevance to a global solution

Preliminaries

- Minkowski sum and PD
  - $P \oplus -Q = \{ p-q \mid p \in P, q \in Q \}$
  - PD := minimum distance between O_{Q-P} and the surface of $P \oplus -Q$



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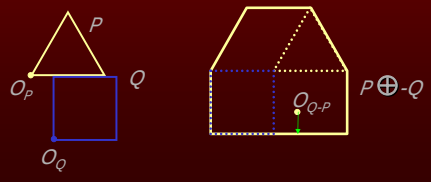

Preliminaries

- Decomposition property of Minkowski sum
  - If $P = P_1 \cup P_2$, *then* $P \oplus Q = (P_1 \oplus Q) \cup (P_2 \oplus Q)$
- Computing Minkowski sum
  - Convex: O(n *log*(n))
    - where n is the number of features
  - Non-Convex: $O(n^6)$ computational complexity
    - In theory, use the convolution or the decomposition property
    - In practice, very hard to implement

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PD Algorithm : A Hybrid Approach

- $P \oplus Q = (P_1 \oplus Q) \cup (P_2 \oplus Q)$
  - where $P = P_1 \cup P_2$

PD Algorithm : A Hybrid Approach

- $P \oplus Q = (P_1 \oplus Q) \cup (P_2 \oplus Q)$
  - where $P = P_1 \cup P_2$
- **Precomputation: Decomposition**

PD Algorithm : A Hybrid Approach

- $P \oplus Q = (P_1 \oplus Q) \cup (P_2 \oplus Q)$
  - where $P = P_1 \cup P_2$
- Precomputation: Decomposition
- Runtime:
  - **Object Space: Pairwise Minkowski sum computation**

PD Algorithm : A Hybrid Approach

- $P \oplus Q = (P_1 \oplus Q) \cup (P_2 \oplus Q)$
  - where $P = P_1 \cup P_2$
- Precomputation: Decomposition
- Runtime:
  - Object Space: Pairwise Minkowski sum computation
  - **Image Space: Union by graphics hardware**

[Kim, Otaduy, Lin & Manocha, SCA'01]

PD Computation Pipeline

Precomputation

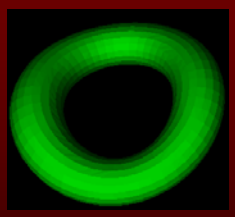
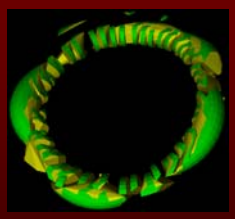
Convex Decomposition

Run-time PD Query

Pairwise Minkowski Sum (Object Space) → Closest Point Query (Image Space)

[Kim, Otaduy, Lin & Manocha, SCA'02]

Convex Surface Decomposition

- [Ehmann and Lin '01]
- Decompose an object into a collection of convex surface patches
- Compute the convex hull of each surface patch

Pairwise Minkowski Sum

- Algorithms
  - Convex hull property of convex Minkowski sum
    - $P \oplus Q = \mathbf{ConvHull}\{\mathbf{v}_i + \mathbf{v}_j \mid \mathbf{v}_i \in V_P, \mathbf{v}_j \in V_Q\}$, where P and Q are convex polytopes
  - Topological sweep on Gauss map [Guibas '87]
  - Incremental surface expansion [Rossignac '92]

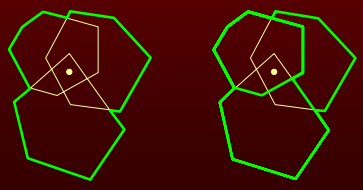
Closest Point Query

Goal

- Given a collection of convex Minkowski sums, compute the shortest distance from the origin to the surface of their union
- An exact solution is computationally expensive ⇒ Approximation using graphics hardware

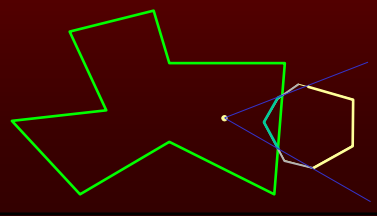
Closet Point Query

- Main Idea
  - Incrementally expand the current front of the boundary



Closest Point Query

1. Render front faces, and open up a window where z-value is less than the current front
2. Render back faces w/ z-greater-than test
3. Repeat the above *m* times, where m := # of obj's



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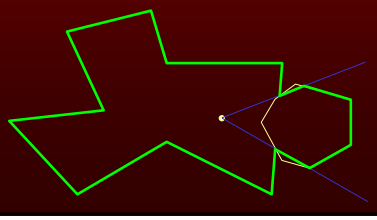

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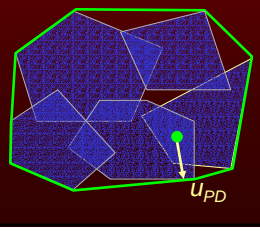
Motivation

- PD is shallow in practice
- Convex decomposition has O(n) convex pieces in practice

→ Culling strategy is suitable and very effective

Object Space Culling

- Basic Idea
  - If we know the upper bound on PD, u_{PD}, we do not need to compute the Mink. sum of pairs whose Euclidean dist is more than u_{PD}



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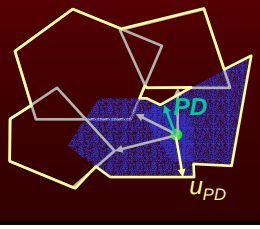

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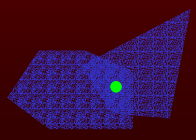
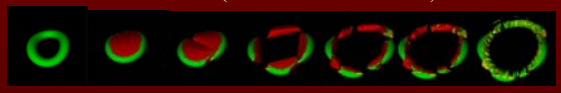


Image Space Culling

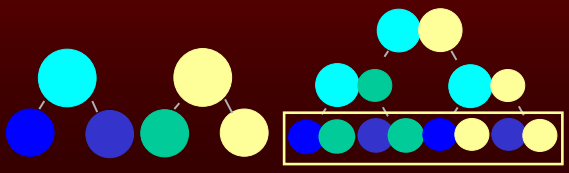
- Rendering only once for the Minkowski sums containing the origin
- Refine the upper bound for every view frustum
- View frustum culling

Hierarchical Culling

- BVH Construction (Ehmann and Lin '01)

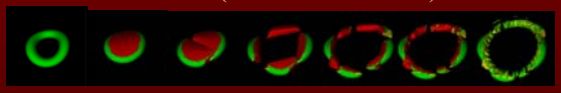


- Hierarchical Culling

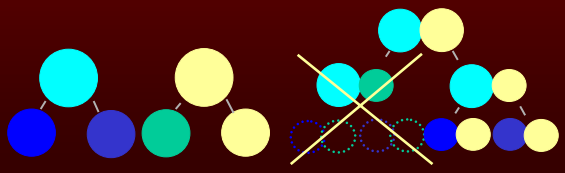


Hierarchical Culling

- BVH Construction (Ehmann and Lin '01)



- Hierarchical Culling



Hierarchical Refinement

Precomputation

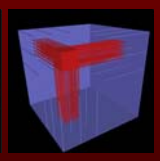
Convex Decomposition → BVH Construction

Run-time PD Query

Culling → Pairwise Minkowski Sum → Closest Point Query

Refine the current PD estimate, and go to the next level of BVH

PD Benchmarks

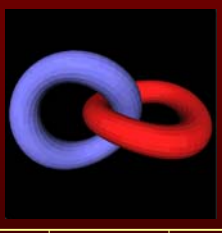
Touching Tori	Interlocked Tori	Interlocked Grates	Touching Alphabets
			
0.3 sec (4 hr)	3.7 sec (4 hr)	1.9 sec (177 hr)	0.4 sec (7 min)

• With Accel. (Without Accel.)

PD Benchmarks

Models	Tri	Convex Pieces	PD w/o Accel.	PD w/ Accel
Touching Tori	2000	67	4 hr	0.3 sec
Interlocked Tori	2000	67	4 hr	3.7 sec
Interlocked Grates	444, 1134	169, 409	177 hr	1.9 sec
Touching Alphabets	144,152	42, 43	7 min	0.4 sec

Hierarchical Culling Example



Level	Cull Ratio	Mink Sum	HW Query	PD_{est}
3	31.2%	0.219 sec	0.220 sec	0.99
5	96.7%	0.165 sec	0.146 sec	0.53
7	98.3 %	1.014 sec	1.992 sec	0.50

Implementation

- SWIFT++ [Ehmann and Lin '01]
- QHULL
- OpenGL for closest point query

Implementation

```
void DrawUnionOfConvex(ConvexObj *ConvexObjs, int NumConvexObjs)
{
  glClearDepth(0);
  glClearStencil(0);
  glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT | GL_STENCIL_BUFFER_BIT);

  glEnable(GL_DEPTH_TEST);
  glEnable(GL_STENCIL_TEST);
  for (int i=0; i<NumConvexObjs; i++)
    for (int j=0; j<NumConvexObjs; j++)
    {
      glDepthMask(0);
      glColorMask(0,0,0,0);
      glDepthFunc(GL_LESS);
      glStencilFunc(GL_ALWAYS,1,1);
      glStencilOp(GL_KEEP,GL_REPLACE,GL_KEEP);
      ConvexObjs[j].DrawFrontFaces();

      glDepthMask(1);
      glColorMask(1,1,1,1);
      glDepthFunc(GL_GREATER);
      glStencilFunc(GL_EQUAL,0,1);
      glStencilOp(GL_ZERO,GL_KEEP,GL_KEEP);
      ConvexObjs[j].DrawBackFaces();
    }
  }
```

Accuracy of PD computation

- Our algorithm computes an upper bound to PD
- Image space computation determines the tightness of the upper bound
  - Pixel resolution
  - Z-buffer precision
- In practice, with 256×256 pixel resolution, the algorithm rapidly converges to the PD

Overview

- Preliminaries
  - Minkowski sum-based Framework
- Basic PD Algorithm: A Hybrid Approach
- Acceleration Techniques
  - Object Space Culling
  - Hierarchical Refinement
  - Image Space Culling
- **Application to Rigid Body Dynamic Simulation**

Application to Rigid Body Simulation

- Interpenetration is often unavoidable in numerical simulations
- Need for a consistent and accurate measure of PD
- Penalty-based Method
  - $F = (k \bullet d)\boldsymbol{n}$
    - d: PD, $\boldsymbol{n}$: PD direction, k:stiffness constant

Application to Rigid Body Simulation

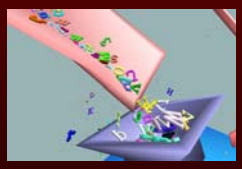
- Time Stepping Method
  - Estimate the time of collision (TOC)
    - s: separation dist before interpenetration
    - d: PD, $\boldsymbol{n}$: PD dir after interpenetration
    - $\boldsymbol{v}_s$: relative velocity of closest features
    - $\boldsymbol{v}_d$: relative velocity of PD features

Application to Rigid Body Simulation

- $x(t)$: 1D distance function between closest features and PD features projected to PD direction
- $x(0) = s$, $x(T) = d$
- $dx/dt(0) = \boldsymbol{v}_s \cdot \boldsymbol{n}$, $dx/dt(T) = \boldsymbol{v}_d \cdot \boldsymbol{n}$
- Compute the roots of $x(t) = 0$

Rigid Body Simulation Demo

Example 1
Average complexity: 250 triangles
60 Convex Pieces / Object



Example 2
Average complexity: 250 triangles
200 letters and alphabets

Rigid Body Simulation Demo

Summary

- First practical PD algorithm using a hybrid approach
- Acceleration techniques:
  - Object space culling
  - Image space culling
  - Hierarchical refinement
- Application to rigid body simulation

Proximity Queries

- **Object-Space Methods**
  - Considerable pre-processing for hierarchy construction
  - Difficult to achieve real-time performance on complex deformable and/or breaking models
- **Image-Space Techniques**
  - Primitive rasterization ↔ sorting in screen-space
  - Applicable to interference tests

Problems with Image-Based Methods

- Closed models
- Frame buffer readbacks – slow
  - NVIDIA GeForce 4, Dell Precision Workstation with 1Kx1K depth buffer taking 50ms

Goals

- Interactive Performance
- Complex objects
  - Large number of objects
  - High primitive count
  - Non-convex objects
  - Open and closed objects
- Non-Rigid Motion
  - Deformable bodies
  - Changing topology

Overview

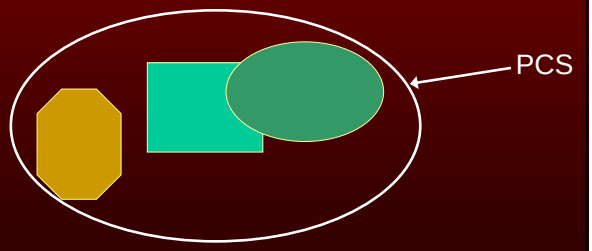
- **Potentially Colliding Set (PCS) computation**
- **Exact collision tests on the final PCS**

GPU based PCS computation

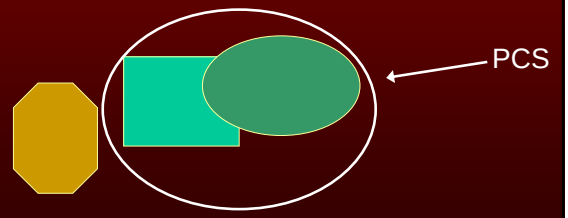
Using CPU

Potentially Colliding Set (PCS)

- A PCS consists of the set of objects that can potentially collide with each other

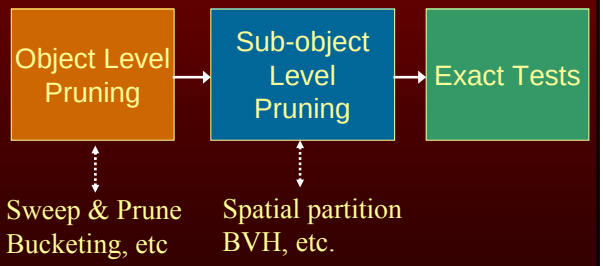


Potentially Colliding Set (PCS)



Comparison

- **Object Level Pruning ↔ Broad Phase**
- **Sub-object Level Prunning ↔ Narrow Phase**

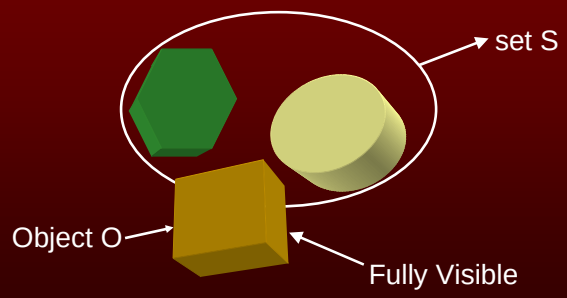


Visibility Computations

Lemma 1: *An object O does not collide with a set of objects S if O is fully visible with respect to S*

→ **Utilize visibility for PCS computation**

Collision Detection using Visibility Computations



PCS Pruning

Lemma 2: *Given n objects $O_1,O_2,\ldots,O_n$, an object O_i does not belong to PCS if it does not collide with $O_1,\ldots,O_{i-1},O_{i+1},\ldots,O_n$*

→ **Prune objects that do not collide**

PCS Pruning

O_1 O_2 … O_{i-1} O_i O_{i+1} … O_{n-1} O_n

PCS Pruning

O_1 O_2 … O_{i-1} O_i

PCS Pruning

O_i O_{i+1} … O_{n-1} O_n

PCS Computation

- Each object tested against all objects but itself
- Naive algorithm is $O(n^2)$
- Linear time algorithm
  - Uses two pass rendering approach
  - Conservative solution

PCS Computation: First Pass

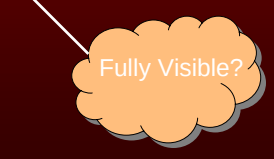
Render →

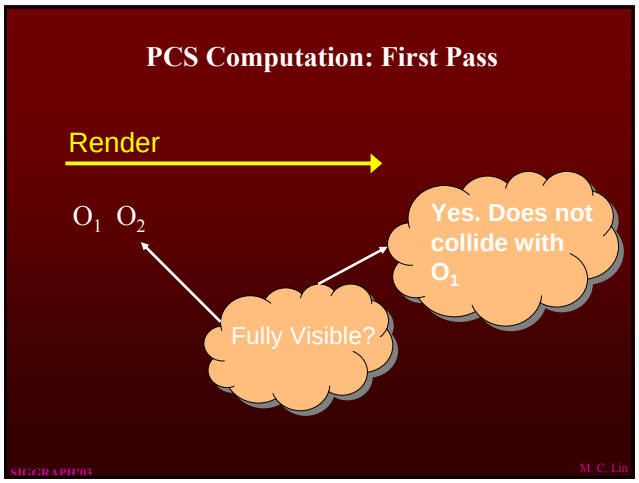
O_1 O_2 … O_{i-1} O_i O_{i+1} … O_{n-1} O_n

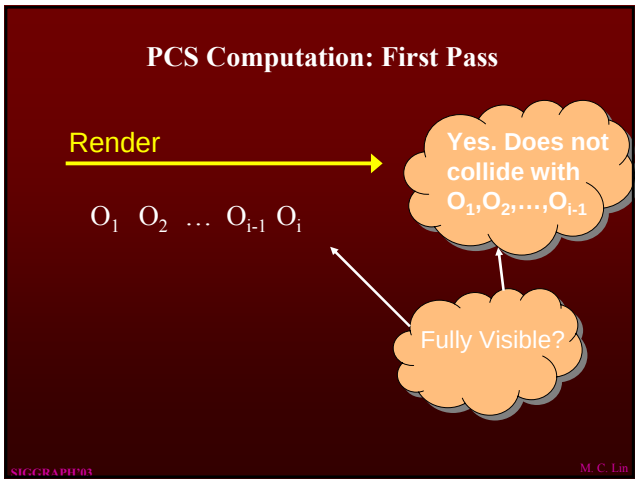
PCS Computation: First Pass

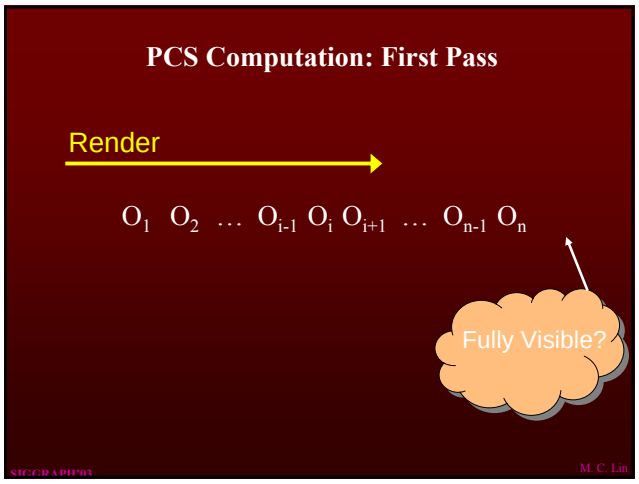
Render →

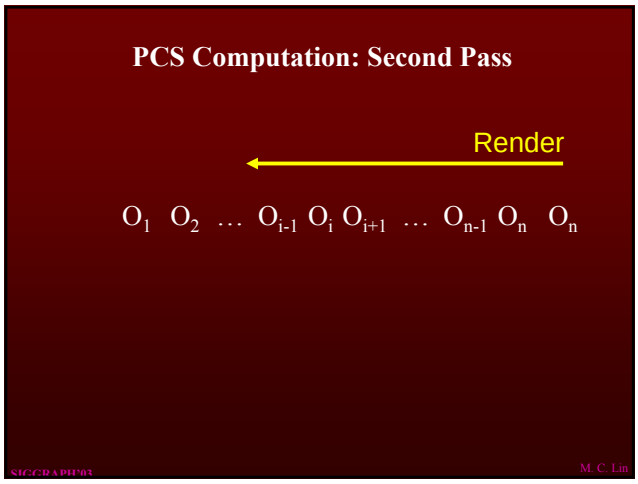
O_1

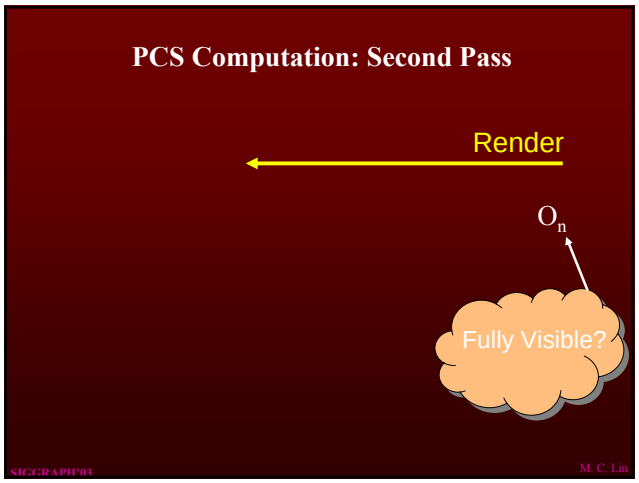


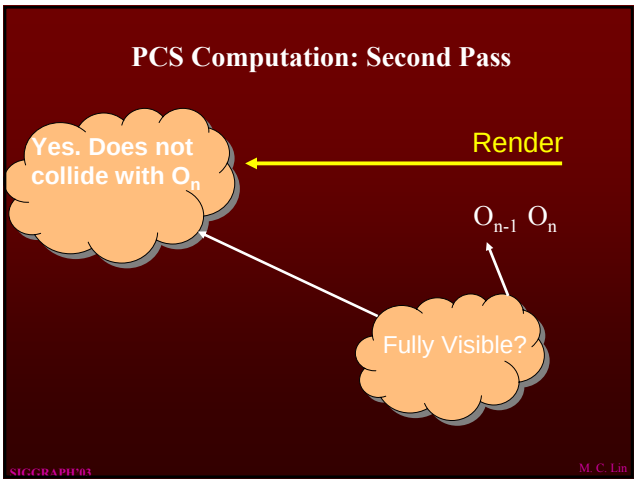

PCS Computation: First Pass
Render
O_1 O_2
Yes. Does not collide with O_1
Fully Visible?
SIGGRAPH'03
M. C. Lin

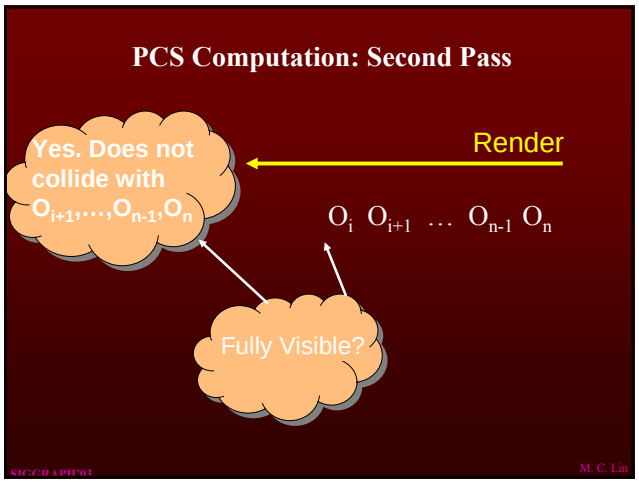

PCS Computation: First Pass
Render
O_1 O_2 … O_{i-1} O_i
Yes. Does not collide with $O_1,O_2,\ldots,O_{i-1}$
Fully Visible?
SIGGRAPH'03
M. C. Lin

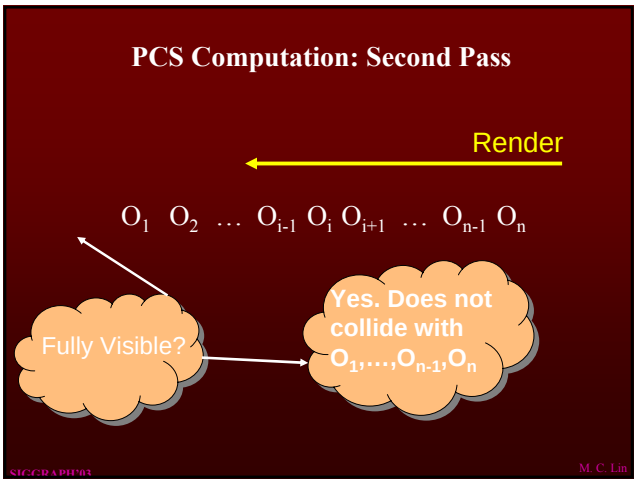

PCS Computation: First Pass
Render
O_1 O_2 … O_{i-1} O_i O_{i+1} … O_{n-1} O_n
Fully Visible?
SIGGRAPH'03
M. C. Lin


PCS Computation: Second Pass
Render
O_1 O_2 … O_{i-1} O_i O_{i+1} … O_{n-1} O_n O_n
SIGGRAPH'03
M. C. Lin

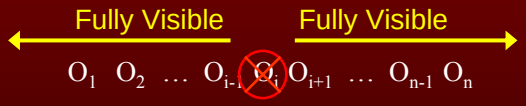

PCS Computation: Second Pass
Render
O_n
Fully Visible?
SIGGRAPH'03
M. C. Lin


PCS Computation: Second Pass
Yes. Does not collide with O_n
Render
O_{n-1} O_n
Fully Visible?
SIGGRAPH'03
M. C. Lin

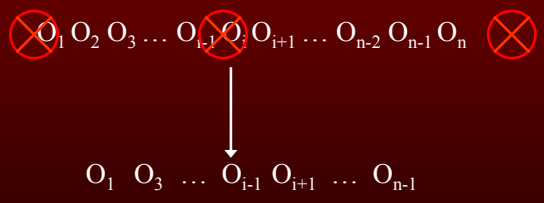

PCS Computation: Second Pass
Yes. Does not collide with $O_{i+1},\ldots,O_{n-1},O_n$
Render
O_i O_{i+1} … O_{n-1} O_n
Fully Visible?
SIGGRAPH'03
M. C. Lin


PCS Computation: Second Pass
Render
O_1 O_2 … O_{i-1} O_i O_{i+1} … O_{n-1} O_n
Fully Visible?
Yes. Does not collide with $O_1,\ldots,O_{n-1},O_n$
SIGGRAPH'03
M. C. Lin

PCS Computation



PCS Computation



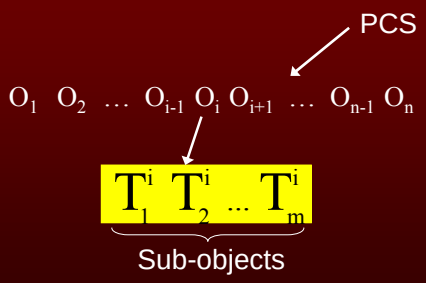
Sub-Object Level: Overlap Localization

- Each object is composed of sub-objects
- We are given n objects $O_1,\ldots,O_n$
- Compute sub-objects of each object O_i that overlap with sub-objects of other objects

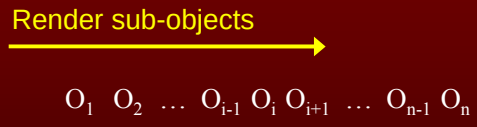
Sub-Object Level: Overlap Localization

- Our solution
  - Test if each sub-object of O_i overlaps with sub-objects of $O_1,..O_{i-1}$
  - Test if each sub-object of O_i overlaps with sub-objects of $O_{i+1},...,O_n$
- Linear time algorithm
- Extend the two pass approach

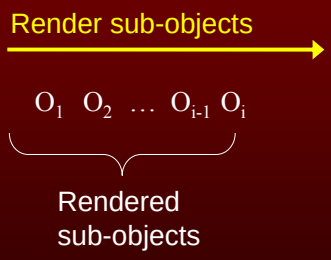
Sub-Object Level: Overlap Localization



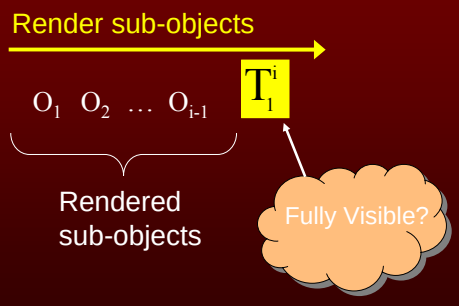
Sub-Object Overlap Localization: First Pass

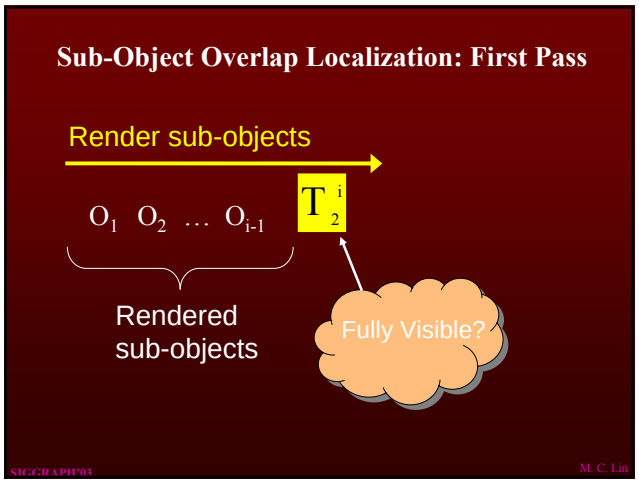


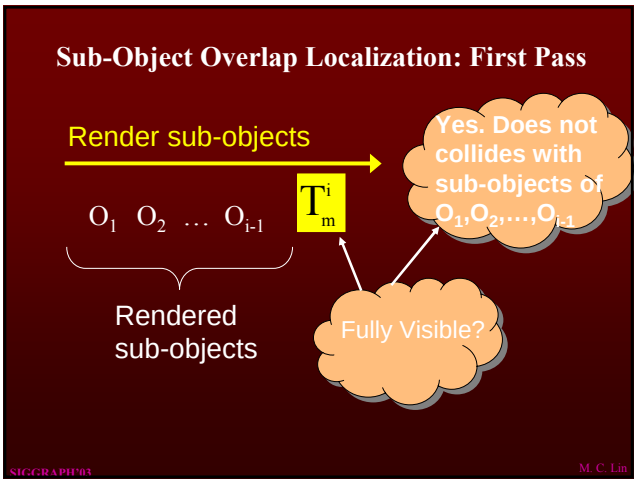
Sub-Object Overlap Localization: First Pass

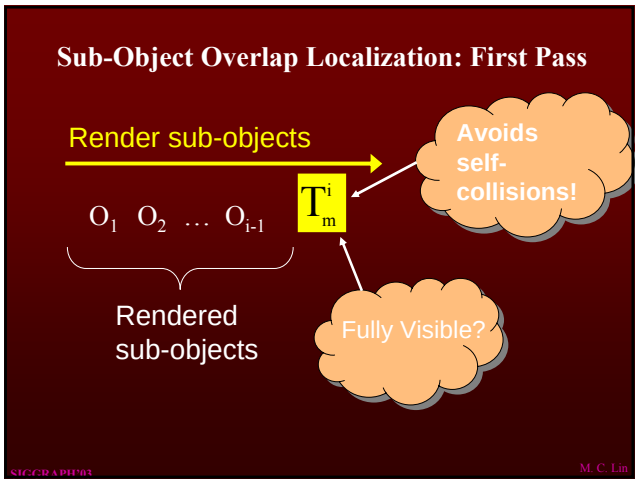


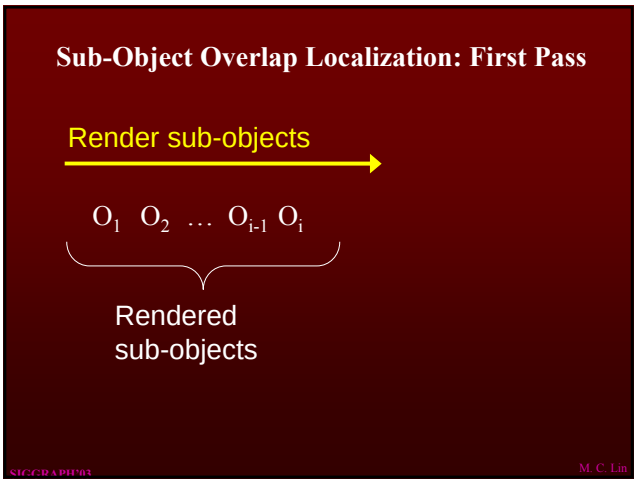
Sub-Object Overlap Localization: First Pass




Sub-Object Overlap Localization: First Pass
Render sub-objects
O_1 O_2 … O_{i-1}
T_2^i
Rendered sub-objects
Fully Visible?
SIGGRAPH'03
M. C. Lin


Sub-Object Overlap Localization: First Pass
Render sub-objects
O_1 O_2 … O_{i-1}
T_m^i
Yes. Does not collides with sub-objects of $O_1, O_2, \ldots, O_{i-1}$
Rendered sub-objects
Fully Visible?
SIGGRAPH'03
M. C. Lin


Sub-Object Overlap Localization: First Pass
Render sub-objects
O_1 O_2 … O_{i-1}
T_m^i
Avoids self-collisions!
Rendered sub-objects
Fully Visible?
SIGGRAPH'03
M. C. Lin


Sub-Object Overlap Localization: First Pass
Render sub-objects
O_1 O_2 … O_{i-1} O_i
Rendered sub-objects
SIGGRAPH'03
M. C. Lin

Sub-Object Overlap Localization: First Pass

O_1 O_2 … O_{i-1} O_i O_{i+1} … O_{n-1} O_n

Rendered sub-objects

Sub-Object Overlap Localization: Second Pass

Render sub-objects

O_1 O_2 … O_{i-1} O_i O_{i+1} … O_{n-1} O_n

Sub-Object Level Overlap Localization

O_1 O_2 … O_{i-1} O_i O_{i+1} … O_{n-1} O_n

Sub-objects

Visibility Queries

- We require a query
  - Tests if a primitive is fully visible or not
- Current hardware supports occlusion queries
  - Test if a primitive is visible or not
- Our solution
  - Change the sign of depth function

Visibility Queries

	Depth function	
	GEQUAL	LESS
All fragments	Pass	Fail
	Fail	Pass

Occlusion query

Query not supported

Examples - HP_Occlusion_test, NV_occlusion_query

Bandwidth Analysis

- Read back only integer identifiers
  - Independent of screen resolution

Optimizations

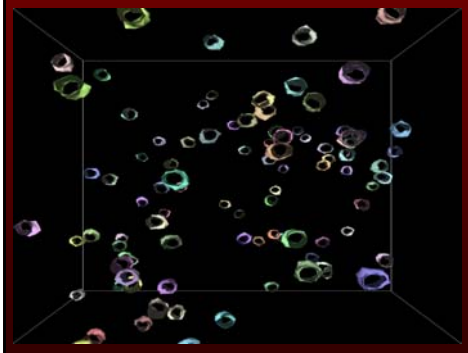
- First use AABBs as object bounding volume
- Use orthographic views for pruning
- Prune using original objects

Implementation

- Dell Precision workstation
- Dell M50 Laptop

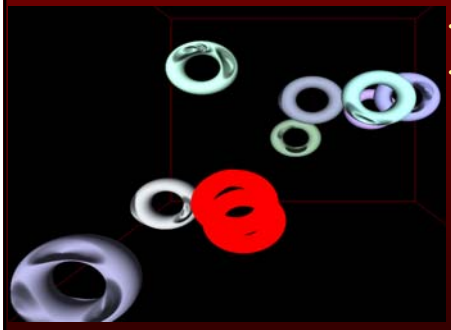
[Govindaraju, Redon, Lin, Manocha, GH'03]

Test Models: Environment 1



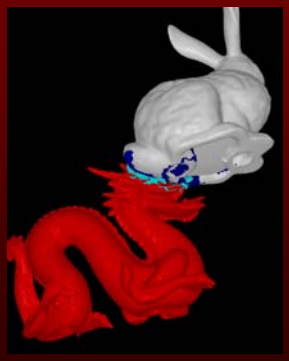
- 100 deforming cylinders
- Total – 20K triangles

Test Models: Environment 2



- Deforming torii
- Each torus – 20K triangles

Test Model: Environment 3



- 250K Dragon
- 35K Bunny

Test Model: Environment 4

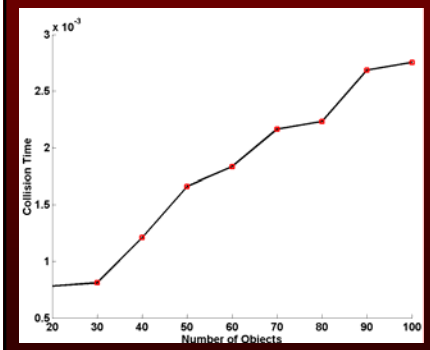
- Breaking dragon – 250K triangles
- Bunny – 35K triangles

[Govindaraju, Redon, Lin, Manocha, GH'03]

Live Demo of Environment 4

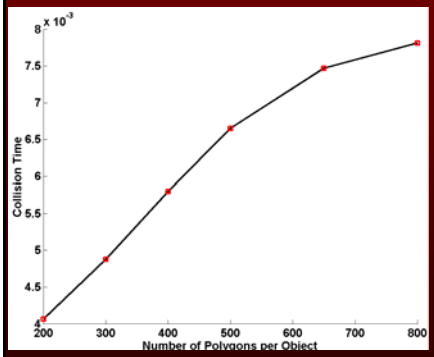
- Dell M50 laptop, 2.4GHz Pentium IV-M CPU, NVIDIA Quadro4 700GoGL GPU, 1GB memory running Windows XP

Performance



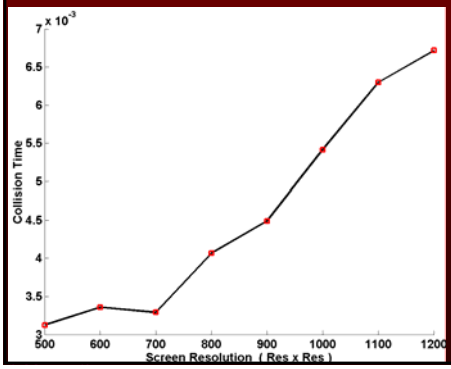
Number of objects v/s collision time

Performance



Number of polygons v/s collision time

Performance



Screen resolution v/s collision time

Advantages

- No coherence
- No assumptions on motion of objects
- Works on generic models
- A fast pruning algorithm
- No frame-buffer readbacks
- Applicable to deformable & breaking models

Limitations

- No distance or penetration depth information
- Resolution limited accuracy
- No self-collision detection

Summary

- First collision detection using GPU
  - Applicable to polygon soups, open boundary, etc.
  - No assumptions on motion
  - Allowing change of topology and dynamic geometry
  - Unified approach for object & sub-object level culling
- Linear time PCS computation algorithm
- No frame buffer readbacks

Organization

- Motion Planning
- Proximity Queries
- **Physically-based Simulation**

Visual Simulation of Ice Growth

Motivation

- Special Effects
- Computer Animation

A scene from X-Men 2

Motivation

- "We built a particle system that produced the effect of fingers of ice … which was tricky, since the particles didn't do that automatically"

  - Arnon Manor, Cinesite

Our approach

- A method for modeling and simulating visually complex structure of ice – both geometrically and optically
  - Physical Simulation using Phase Field Methods
  - Simulation Acceleration using GPUs & Banded Opt.
  - Controlling Complexity
  - Post-Processing for Rendering

[Kim and Lin, SCA'03]

Phase Field Methods

- Temperature PDE

$$\frac{\partial T}{\partial t} = a^2 \nabla^2 T + K \frac{\partial p}{\partial t}$$

- Phase PDE

$$\tau \frac{\partial p}{\partial t} = \nabla \cdot \left(\varepsilon^2 \nabla p\right) - \frac{\partial}{\partial x}\left(\varepsilon \frac{\partial \varepsilon}{\partial \theta}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial y}\left(\varepsilon \frac{\partial \varepsilon}{\partial \theta}\frac{\partial p}{\partial x}\right) + p(1-p)\left(p - \frac{1}{2} + m\right)$$

Ice Morphology

dendritic plate isotropic

GPU Acceleration

- Fields map easily to GPU
- Framework of [Harris et al. 2002]
  - Fields sent as textures
  - Fragment program runs PDE per texel
  - ~100 lines of Cg!

Performance

- Only unbanded currently possible
- No early exit yet on GPU

Grid Size	CPU(Hz)	GPU(Hz)	Speedup
64x64	250	624	2.50x
128x128	25	236	9.44x
256x256	8	67.47	8.43x
512x512	3.5	17.67	5.05x
1024x1024	1.08	3.77	3.49x

Real-Time Capture

[Kim and Lin, SCA'03]

Future Work

- Extension to motion planning of deformable robots and other types of constraints
- PD computation for rotational movement using GPU
- Continuous Collision Detection using GPUs
- Physically-based modeling of paint media, phase transition (melting, solidification, etc)

Collaborators

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The End