Advanced Topic

ARTICULATED BODY DYNAMICS

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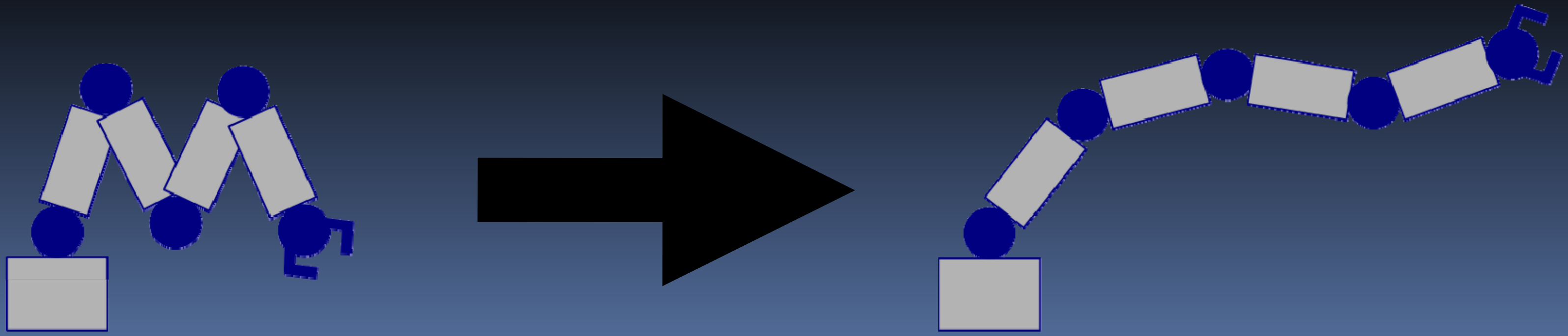
Agenda

- Inverse Dynamics in general
- Efficiency
- A power tool for designing algorithms – Recursion
- Recursive Newton-Euler Algorithm
- Forward Dynamics in general
- Featherstone's Algorithm
- Conclusion
- Reference

Inverse Dynamics (1)

- Given the kinematic representation of motion, inverse dynamics calculates the forces necessary to achieve that motion
- Inverse kinematics will tell us what the motion is
- Inverse dynamics will tell us how to do it



Inverse Dynamics (2)

- The calculation of the forces required at a robot's joints to produce a given set of joint accelerations
- Applications
  - Robot control
  - Trajectory planning
- Rapid execution is essential as it is used heavily for real-time control

Efficiency (1)

- The classic approach to inverse dynamics involved a Lagrangian formulation, which was O(n⁴)
- Non-recursive approaches of either Lagrangian or Newton-Euler formulations result in equations such as:

$$Q_i = \sum_{j=1}^{n} \underset{O(n^2)}{H_{ij}} \ddot{q}_j + \sum_{j=1}^{n} \sum_{k=1}^{n} \underset{O(n^3)}{C_{ijk}} \dot{q}_j \dot{q}_k + g_i$$

Efficiency (2)

- Optimization technique: use recursion
    - Requires a reformulation of the equations
    - Can reduce complexity down to O(n)
    - Reduces computational requirement as well

Recurrence Relations

- Equations defining a member of a sequence in terms of its predecessors
- Example
  - $x_{n+1} = x_n + x_{n-1}$
  - $x_0 = x_1 = 1$
  - Fibonacci sequence

Example: Recurrence Relation

- Let the matrix B be defined as
  - $B = A_1A_2 \ldots A_n$, where A_i is a matrix
- How do we compute the derivative of B
  - Brute force
    - Gets expensive fast
  - Use recursion

Example: Brute Force

- Say $B = A_1A_2A_3A_4A_5$
- Then $B' =$

  $A_1'A_2A_3A_4A_5 +$

  $A_1A_2'A_3A_4A_5 +$

  $A_1A_2A_3'A_4A_5 +$

  $A_1A_2A_3A_4'A_5 +$

  $A_1A_2A_3A_4A_5'$

- Computational requirement is n^2-n matrix multiplications and n-1 matrix additions

Example: Recursion

- Recall $B = A_1 A_2 \ldots A_n$
- Define $B_{i+1} = B_i A_{i+1}$
  - It follows that $B_n = B$, thus $B_n' = B'$
- So

  $B_{i+1}' = B_i' A_{i+1} + B_i A_{i+1}'$
- To calculate B', we start with $B_1' = A_1'$ and iterate up to B_n

Example: Recursion

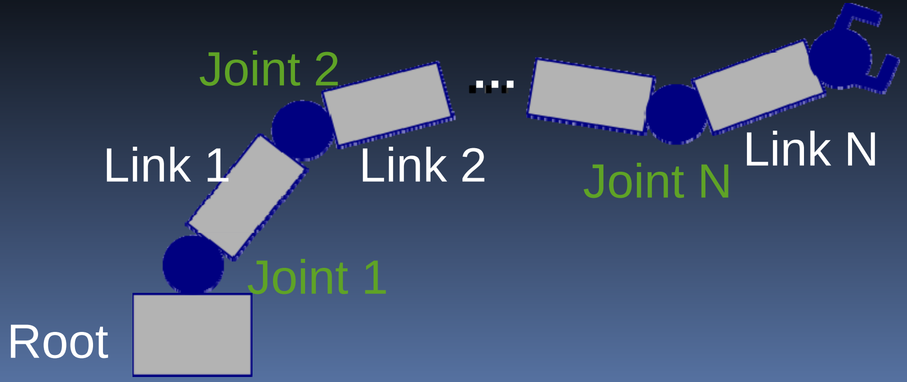
- So we iteratively compute $B_2,\ldots,B_{n-1}$
  - n-2 iterations
  - 1 matrix multiplication
- Then we iteratively compute $B_2',\ldots,B_n'$
  - n-1 iterations
  - 2 matrix multiplications, 1 matrix addition
- Total: 3n-4 matrix multiplications, n-1 additions
  - Non-recursive: n2-n matrix multiplications and n-1 additions

Example: Results

- By using recurrence relations, we were able to reformulate the solution using recursion
- $O(n^2)$ to $O(n)$ improvement
- We can get much more dramatic results with inverse dynamics
    - $O(n^4)$ to $O(n)$

Robot System Model

- N movable links, labeled 1,…,N (from the root to the terminal)
- A fixed base link, labeled 0
- λ(i) is the parent link of link i
  - Links are numbered so that λ(i) < i
- N joints, where joint i connects link λ(i) to link i



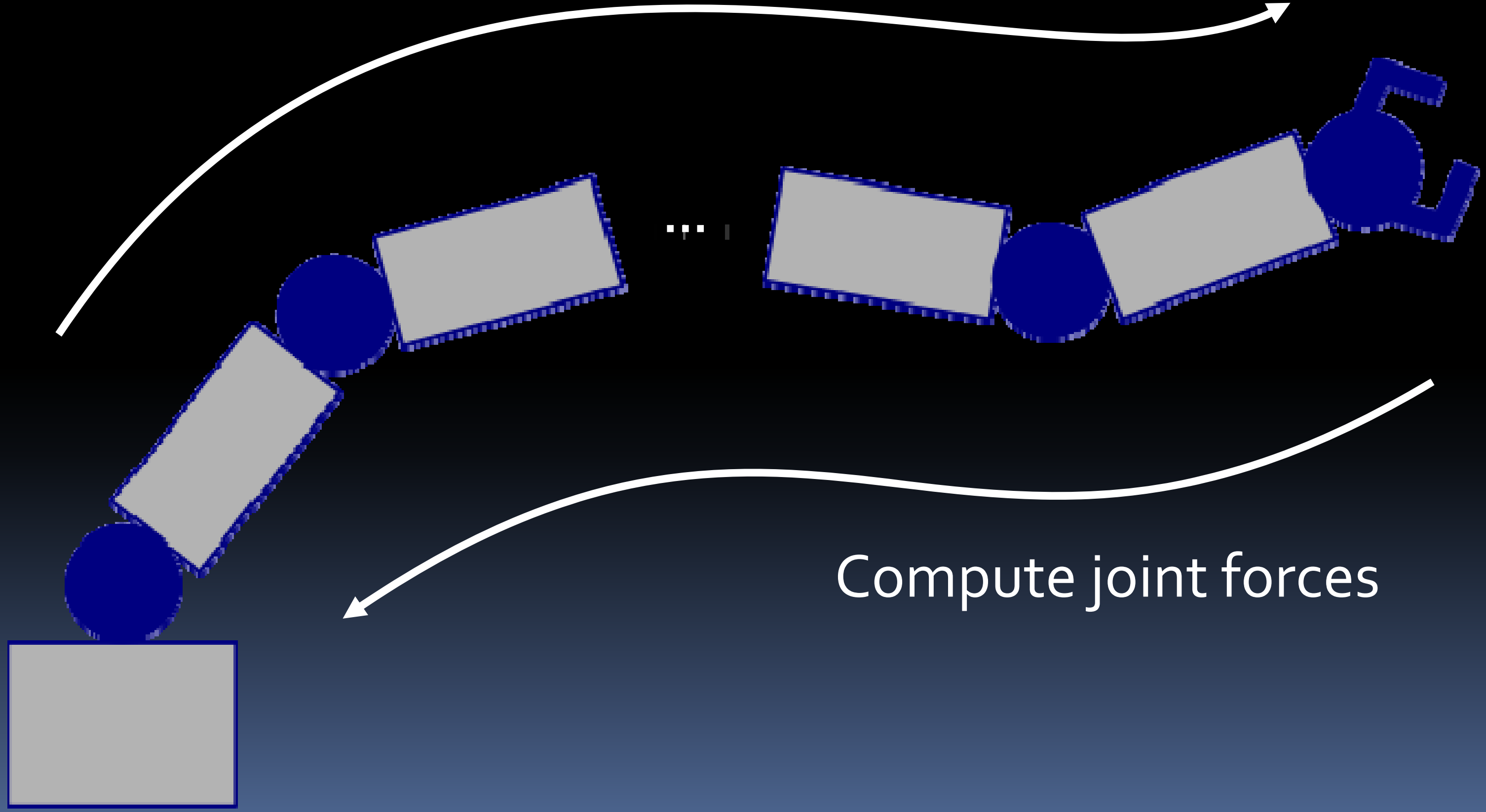
Recursive Newton-Euler Algorithm (RNEA)

- The most efficient currently known general method for calculating inverse dynamics
- Input: a system model of a robot and the values of the desired joint accelerations
- Output: the joint forces required to produce the desired joint accelerations

RNEA Steps

- Step 1: calculate the velocity and acceleration of each link i
- Step 2: calculate the net force acting on each link from its motion and inertia
- Step 3: calculate the joint forces required to produce the forces in step 2

RNEA



Compute velocity, acceleration of links

Compute net forces

Compute joint forces

Step 1: Link Motion

- v_i – absolute velocity of link i
- a_i – absolute acceleration of link I
- $s_i\,\dot{q}_i$ – velocity across link i (relative velocity of link i with respect to link i-1
- Recurrence relation:
- $v_i = v_{i-1} + s_i\,\dot{q}_i \qquad (v_0 = 0)$
- $a_i = a_{i-1} + v_i \times s_i\,\dot{q}_i + s_i\,\ddot{q}_i \qquad (a_0 = 0)$

Step 2: Net Forces on the Link

- The net force on link i is given by the link's rate of change of momentum

$$f_i^* = \frac{d(I_i V_i)}{dt} = I_i a_i + v_i \times I_i v_i$$

Step 3: Joint Forces

- First we must find the total force transmitted from link i-1 to link i through joint i
- $$f_i - f_{i+1} = f_i^*$$
- Rearranging, we get
- $$f_i = f_{i+1} + f_i^* \qquad \left(f_n = f_n^*\right)$$

Taking into Account External Forces…

- The equation for joint force becomes
- $f_i = f_{i+1} + f_i^* - f_i^x$
- If you want to model gravity, you can apply a gravitational force to each joint
- It is more efficient, however, to give the robot's base an acceleration of -g (a_o = -g)

RNEA: Pseudocode

```
$v_0 = 0$
$a_0 = 0$
for i = 1 to N do
    $v_i = v_{\lambda(i)} + s_i q_i'$
    $a_i = a_{\lambda(i)} + v_i x s_i q_i' + s_i q_i''$
```

Step 1

```
    $f_i* = I_i a_i + v_i x I_i v_i$
end
```

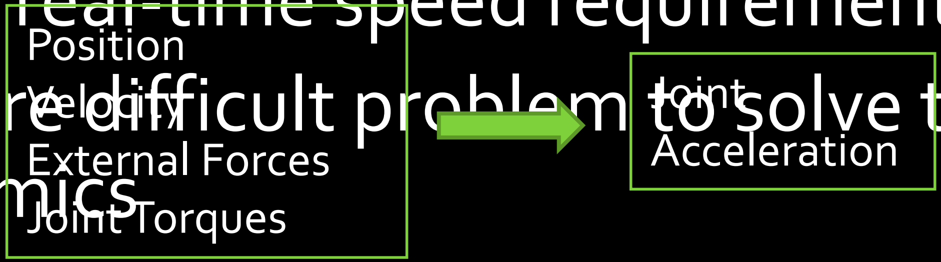
Step 2

```
$f_n = f_n*$
for i = N to 1 do
    $F_{\lambda(i)} = f_i + f_{\lambda(i)}* - f_i^x$
end
```

Step 3

Forward Dynamics

- Primarily for simulation, not necessary to meet real-time speed requirement
- A more difficult problem to solve than inverse dynamics



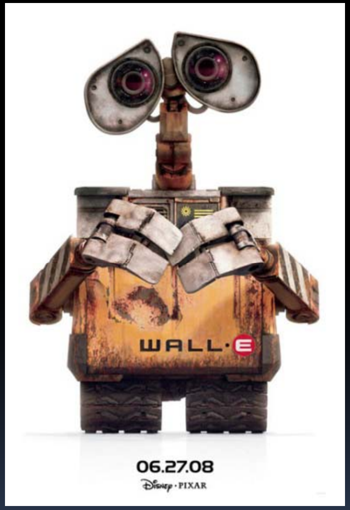
Position
Velocity
External Forces
Joint Torques

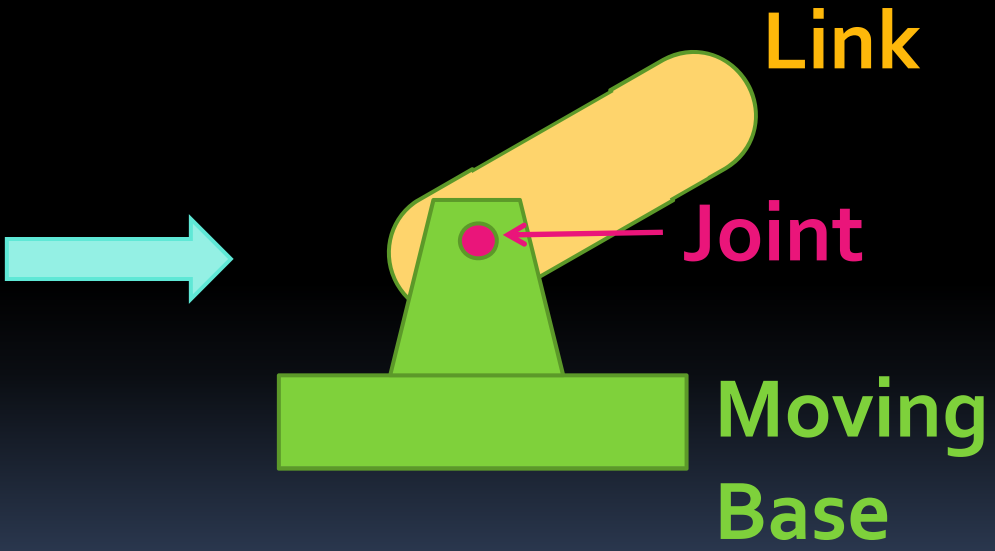
Joint
Acceleration

- Two approaches:
  - Solve the problem directly by calculating recursion coefficients. Ex: Featherstone's Algorithm
  - Obtain and then solve a set of simultaneous equations. Ex: Composite Rigid Body Algorithm

$$\ddot{q} = M^{-1}F$$

Featherstone's Algorithm: Basic Idea



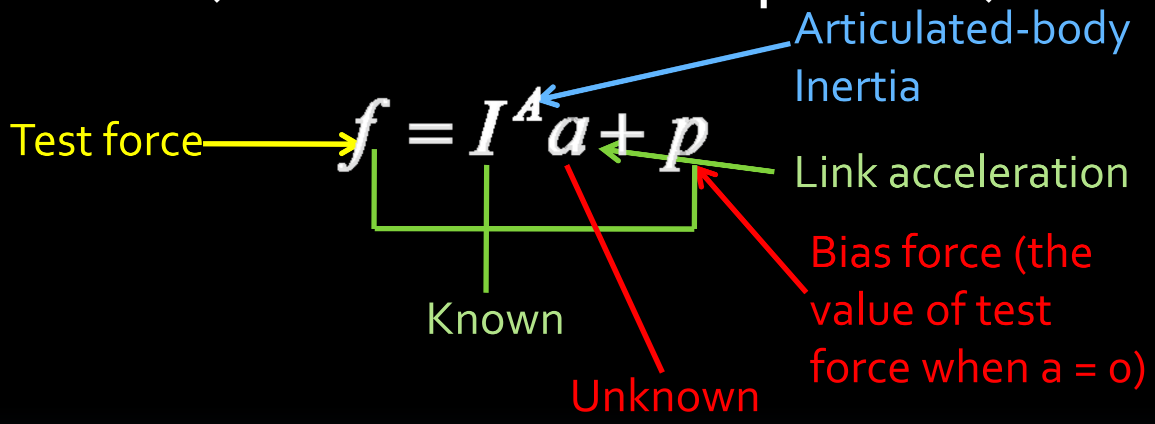


The Characteristics of Featherstone's Algorithm

- Also called "Articulated-Body Algorithm"
- Developed for solving forward dynamics problems
- First version only worked for joints with single degree-of-freedom.
- Second version included a general joint model and was faster.
- Complexity: O(n), faster than CRBA for N>9

Featherstone's Algorithm

- Linear relation between the acceleration and the force (Newton Euler equation):



$$f = I^A a + p$$

Test force

Articulated-body Inertia

Link acceleration

Bias force (the value of test force when a = 0)

Known

Unknown

- No kinematic connection with ground, for ex: a floating system
- If there are external forces such as gravity, the equation becomes $f + f^E = I^A a + p$

Featherstone's Algorithm: Inertia & Bias forces (1)

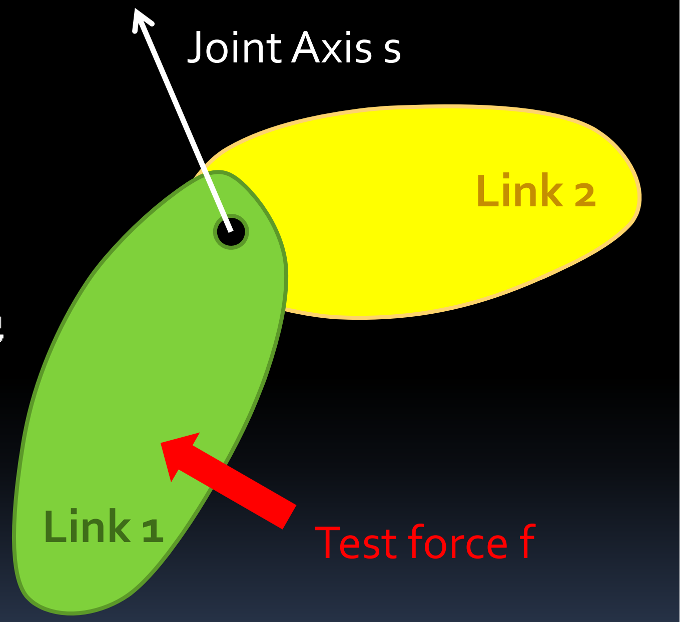
f = net force on link 1 + force transmitted to link 2 through the joint

$$= f_1 + f_2$$

$$= I_1 a_1 + p_1 + I_2 a_2 + p_2$$

$$= I_1 a_1 + v_1 \times I_1 v_1 + I_2 a_2 + p_2 + v_2 \times I_2 v_2$$

… (I)



Constraint imposed by the joint:

$a_2 = a_1 + v_1 \times v_2 + s\alpha$ … (II)

Scalar joint acceleration

Active joint force:

$s^T f_2 = Q$ … (III)

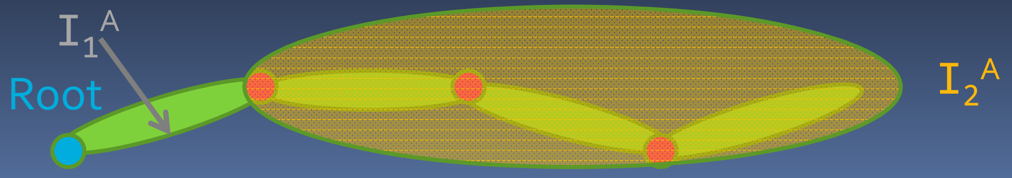
Active joint force

Featherstone's Algorithm: Inertia & Bias forces (2)

From (I), (II), and (III),

$$f = \underbrace{\boxed{\left(I_1 + I_2 - \frac{I_2 s s^T I_2}{s^T I_2 s} \right)}}_{I_1^A} a_1 + \underbrace{\boxed{v_1 \times I_1 v_1 + v_2 \times I_2 v_2 + I_2 \left(v_1 \times v_2 + s \frac{Q - s^T \left(I_2 v_1 \times v_2 + v_2 \times I_2 v_2 \right)}{s^T I_2 s} \right)}}_{p_1}$$

The above derivation is for a system with 2 links only. They can be generalized to multiple links by considering the following scenario:



Featherstone's Algorithm: Inertia & Bias forces (3)

I_1 is still a simple rigid body but I_2 becomes an articulated body. By a similar derivation, we can find out the following recursions for the inertia and the bias force:

Use it's child's information. So if we can start from the terminal link....

$$I_i^A = I_i + \boxed{I_{i+1}^A} - \frac{I_{i+1}^A s_{i+1} (s_{i+1})^T I_{i+1}^A}{(s_{i+1})^T I_{i+1}^A s_{i+1}}$$

$$p_i = v_i \times I_i v_i + \boxed{p_{i+1}} + I_{i+1}^A v_{i+1} \times s_{i+1} \dot{q}_{i+1} + \frac{I_{i+1}^A s_{i+1} \left(Q_{i+1} - (s_{i+1})^T \left(I_{i+1}^A v_{i+1} \times s_{i+1} \dot{q}_{i+1} + p_{i+1} \right) \right)}{(s_{i+1})^T I_{i+1}^A s_{i+1}}$$

Featherstone's Algorithm: Joint acceleration (1)

Definition of joint velocity:

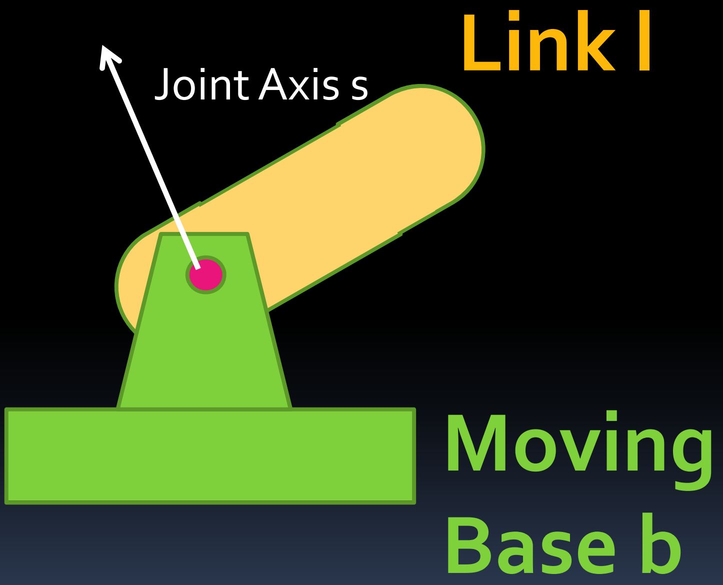
$$v_l - v_b = s\dot{q} \quad \ldots(IV)$$

Take derivatives:

$$a_l - a_b = v_b \times s\dot{q} + s\ddot{q} \quad \ldots(V)$$

Force f applied through the joint:

$$s^T f = Q \quad \ldots(VI)$$



Featherstone's Algorithm: Joint acceleration (2)

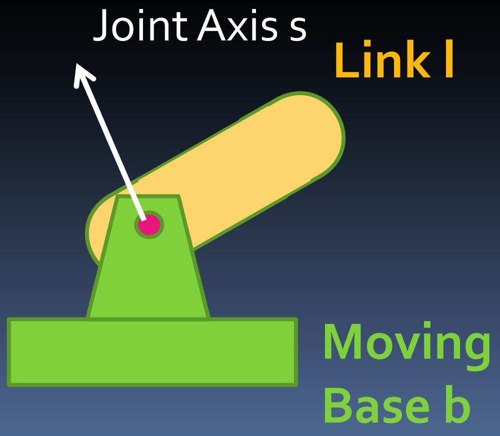
From (IV), (V), (VI), and our linear relationship

$$f = I^A a_l + v_l \times I^A v_l$$

We can obtain

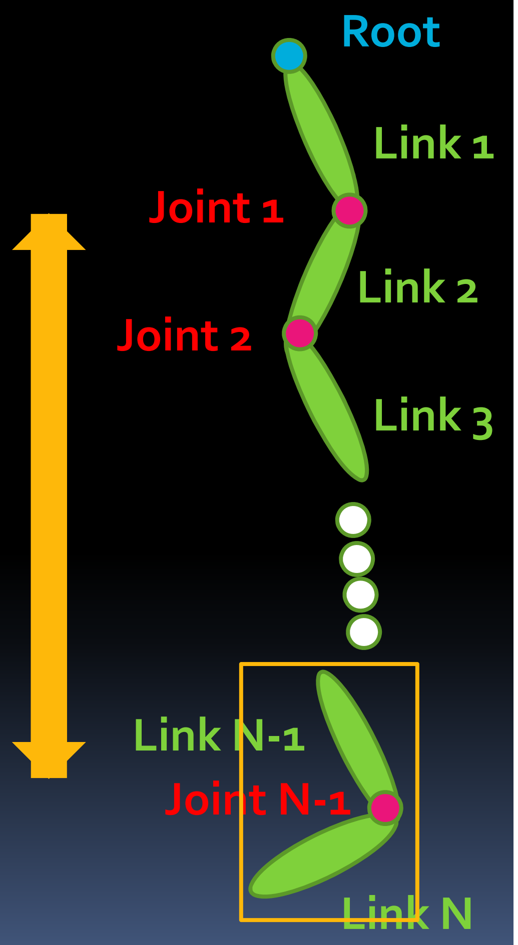
$$\ddot{q} = \frac{Q - s^T\left(I^A\left(a_b + v_b \times s\,\dot{q}\right) + p\right)}{s^T I^A s}$$

$$= \varphi(a_b, v_b, a_l, v_l, I^A, s, \dot{q})$$



Featherstone’s Algorithm: Put all steps together

```
FUNCTION ABA_acceleration(q, q_dot, s, Q) {
        v(1) = 0;
        // Compute velocity for all links.
        FOR link_i=2 TO N
                v(link_i) = p_link(v(link_i))+s*q_dot();
        // Compute the inertia and the bias forces.
        FOR  link_i=N TO 1 {
                compute_I(link_i, link_i-1);
                compute_p(link_i , link_i-1);
        }
        // Compute acceleration for all the joints.
        a(1) = 0; // link 1’s acceleration
        FOR joint_i=1 TO N-1 {
                compute_q_dotdot(joint_i);
                // Compute link acceleration for the next joint.
                a(joint_i+1) = a(joint_i)+v(joint_i).cross(s(joint_i))*q_dot(joint_i)+
                                s(joint_i)*q_dot_dot(joint_i);
        }
}
```



Conclusions

- Inverse Dynamics Vs. Forward Dynamics
- Recursive Newton-Euler Algorithm for solving Inverse Dynamics problems
- Featherstone's Algorithm for solving Forward Dynamics issues
- Use the recursion trick to make your program faster.

References

Highly recommend!

- Roy Featherstone, “Robot Dynamics Algorithm,” Kluwer Academic Publishers 1987
- Featherstone & Orin, “Robot Dynamics: Equations and Algorithms, “ ICRA 2000
- Karen Liu, “Articulated Rigid Bodies,” slides from CS7496/4496 Computer Animation class at Georgia Tech