COMP 790-058
Robot Motion Planning and Multi-Agent Simulation: Fall 2013
HOMEWORK #1 – Due date: Sep. 23, 2013

Notes:
1. You are allowed to discuss the questions. But please write your own answer separately, as opposed to copying someone's answer.
2. You are allowed to make reasonable assumptions for any questions. Please state them carefully.

Problem 1 (Configuration space of lines):
(10 points)

1. Consider an infinite straight line L that can move freely in 3D space. We assume that the line has no distinguishable orientation from one end to the other. Hence, the configuration of L reached by rotating the line by π around a vector perpendicular to L is indistinguishable from the start configuration.

a. What is the number of dimensions of the configuration space C of L?

b. Propose two parameterizations of C (charts): one that makes use of angles, and another that does not make use of angles.

Problem 2 (Convexity in workspace and C-space)):
(20 points)

Let **A** be a robot and **B** be a static obstacle in a 3-D workspace **W**. **A** is made of a single rigid body that can only translate. Its configuration is represented by (x,y,z), the coordinates of a reference point selected in **A** relative to the coordinate system of **W**. Both **A** (at any configuration) and **B** are convex subsets of **W**. Prove that the C-obstacle corresponding to **B** is a convex region of the C-space of **A**.

[Hint: Consider a set $S \subset \mathbf{R}^n$. S is convex if for any two points X and Y of S described by their n coordinates, the point $\lambda X + (1-\lambda)Y$ (with $0 < \lambda < 1$) is in S.]

Problem 3 (Connectedness of a C-obstacle):

(20 points)

A robot **A** is modeled by an oriented line segment of length 4 that can move freely in the plane **W**. The configuration of **A** is described by (x,y,θ), where (x,y) are the coordinates of the center-point of **A** in the coordinate system of **W** and $\theta \in [0,2\pi)$ is the angle between the x-axis of this coordinate system and the oriented line segment representing **A**. Here, the orientation of the line segment allows us to distinguish between the configurations (x,y,θ) and $(x,y,\theta+\pi)$.

The workspace contains a single obstacle **B**, a square centered at (0,2) whose sides have length 2.

1. Draw the C-obstacles corresponding to **B** when **A** is only allowed to translate at fixed orientations $\theta = 0$, $\pi/4$, and $\pi/2$. (Draw three C-obstacles, one for each orientation.)

2. Describe with a few itemized sentences how the C-obstacle corresponding to **B** looks like in the 3D C-space of **A** (that is, when **A** translates and rotates). In particular, is the C-obstacle connected (i.e., made of one single piece)? Does it contain holes? Is it convex? At which orientations its cross-section undergoes qualitative changes? Is there a repeating pattern along the orientation axis?

3. We now constrain the center-point of **A** to remain on the x-axis of the coordinate system of **W**. **A** translates and rotates with this constraint. What is the configuration space of **A**? Draw (approximately) the C-obstacle corresponding to **B**. Is it connected? How does this C-obstacle relate to the one described in Question 2?

Problem 4 (Representation of the configuration space of a robot arm):

(25 points)

Consider a planar robot arm **A** with two revolute joints. Let θ_1 and θ_2 be the two joint angles of **A**.

1. What is the configuration space of **A** in each of the following cases?

- [a] θ_1 and θ_2 take any value in $(-\infty,+\infty)$, that is, the motion at each joint is not limited by any mechanical stop.
- [b] θ_1 and θ_2 take any value in $[0,6\pi]$, that is, the motion at each joint is limited to 3 full rotations.
- [c] θ_1 and θ_2 take any value in $[0,2\pi]$, that is, the motion at each joint is limited to one full rotation.

[d] θ_1 and θ_2 take any value in $[0,\alpha]$, with $\alpha < 2\pi$, that is, none of the joints can perform a full rotation.

2. Suppose that workspace contains an obstacle **B**. How would you represent **A**'s configuration space in cases [a], [b], [c], and [d]?

Problem 5 (Relating distances in workspace and C-space):
(25 points)

Consider a planar robot arm with n sequential links and n revolute joints. Each link is a straight-line segment of length L; one endpoint of the link is called the link's origin, the other the link's extremity. The first joint is at the origin of the first link and is fixed in the workspace. The i^{th} joint ($i = 2, \ldots, n$) coincides with both the extremity of the $(i-1)^{th}$ link and the origin of the i^{th} link. Figure 1 illustrates.

A configuration q of the robot be represented by $(\theta_1,\theta_{2,\ldots,}\ \theta_n)$, where $\theta_1, \ldots, \theta_n$ are the joint angles (their precise definition is not important here). The metric d in the robot's configuration space is the L_∞ metric defined as follows:

For any two configurations $q = (\theta_1,\theta_{2,\ldots,}\theta_n)$ and $q' = (\theta_1',\theta_2'_{,\ldots,}\theta_n')$ we have:
$d(q,q') = \max_{i = 1 \text{ to } n}|\theta_i - \theta_i'|$.

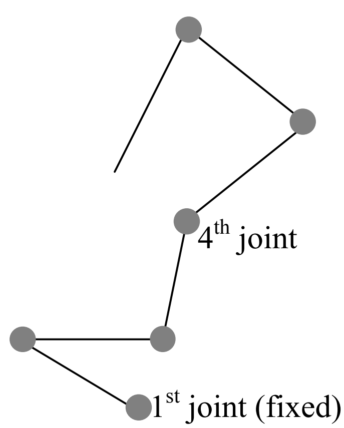

Figure 1: Planar robot arm with n = 6 links

1. Let the robot move from an arbitrary configuration $q = (\theta_1,\theta_{2,\ldots,}\theta_n)$ to another arbitrary configuration $q' = (\theta_1',\theta_2'_{,\ldots,}\theta_n')$ along the straight-line segment joining q and q' in the Cartesian space $\mathbf{R}^n$ (that is, all degrees of freedom are synchronized).

Show that no point of the robot traces a path longer than $\alpha \times d(q,q')$ for some positive constant α. Give a value of α (that necessarily depends on the link's length L and the number n of links, hence is robot-dependent).

2. Let DIST(q,B) be a function that returns the distance between the robot placed at configuration q and a workspace obstacle B (distance between the pair of closest points in the robot and B). Using the result of Question 1, express the radius ρ of the neighborhood

$$N(q) = \{q' \mid d(q,q') \leq \rho\}$$

in which it is guaranteed that the robot can move freely without colliding with B. [Express ρ using α and DIST(q,B).]

3. Let q_1 and q_2 be two configurations of the robot. Propose an algorithm to check the straight-line segment joining q_1 and q_2 in $\mathbf{R}^n$ for collision. Can this algorithm be generalized to other robots? How? Can it be improved? How?