Homework 9

COMP 575/770 Spring 2016

Due: May 2, 2016

Instructions

- Please work on the problems on your own. It is okay to discuss the problems with other students, but please write your answer independently. If you are able to find any part of the solution in a book or some source on the Internet, please acknowledge that source.
- Problem 4 (marked with an asterisk) is optional for COMP 575, and mandatory for COMP 770. COMP 575 students may attempt it for extra credit.

1. Given two endpoints $\mathbf{p}_0$ and $\mathbf{p}_1$, an intermediate point $\mathbf{p}_i$ with its corresponding (but unspecified) parametric variable u_i, and the unit tangent vectors $\mathbf{t}_0$ and $\mathbf{t}_1$, how would you compute a cubic Hermite curve that interpolates the three points and has the specified unit tangent vectors at the boundary?

2. Consider a cubic Bézier curve in $\mathbb{R}^3$, denoted by $\mathbf{P}(t)$. The curve is specified in terms of four control points, $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$ and $\mathbf{p}_3$. A *cusp* on the curve corresponds to a discontinuity in the unit tangent vector. A necessary condition for the existence of a cusp at $t = t_0$ is:
$$\mathbf{P}'(t_0) = 0 \tag{1}$$
Can $\mathbf{P}(t)$ have a cusp? You may assume that the four control points are neither collinear nor coplanar.

3. Consider a Bézier curve $\mathbf{B}(\mathbf{b}_0, \mathbf{b}_1, ..., \mathbf{b}_n : t)$, specified in terms of the control points. Given $0 \leq \bar{t} \leq 1$, show that the curve can be subdivided into two Bézier curves given by:
$$\mathbf{B}(\mathbf{c}_0, \mathbf{c}_1, ..., \mathbf{c}_n : \frac{t}{\bar{t}}), 0 \leq t \leq \bar{t} \tag{2}$$
and
$$\mathbf{B}(\mathbf{d}_0, \mathbf{d}_1, ..., \mathbf{d}_n : \frac{t - \bar{t}}{1 - \bar{t}}), \bar{t} \leq t \leq 1 \tag{3}$$
where $\mathbf{c}_i = \mathbf{b}_0^i(\bar{t}), \mathbf{d}_i = \mathbf{b}_i^{n-i}(\bar{t}), 0 \leq i \leq n$, and $\mathbf{b}_i^j(\bar{t})$ is computed using de Casteljau's algorithm.

4. (*) A rational Bézier curve is defined as:
$$\mathbf{P}(t) = \sum_{i=0}^{n} \mathbf{b}_i \frac{w_i B_i^n(t)}{\sum_{j=0}^{n} w_i B_j^n(t)} \tag{4}$$
What happens to the curve at $t = 0$ as the weight $w_0 \to 0$? Can you derive a lower degree Bézier representation of the curve if $w_0 = 0$?